NAVAL POSTGRADUATE SCHOOL

MONTEREY, CALIFORNIA

THESIS

A PRELIMINARY SHIP DESIGN MODEL FOR CARGO THROUGHPUT OPTIMIZATION

by

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June 2014

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REPORT DOCUMENTATION PAGE			*Form Approved OMB No. 0704-0188*
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instruction, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188) Washington DC 20503.			
1. AGENCY USE ONLY *(Leave blank)*	**2. REPORT DATE** June 2014	**3. REPORT TYPE AND DATES COVERED** Master's Thesis	
4. TITLE AND SUBTITLE A PRELIMINARY SHIP DESIGN MODEL FOR CARGO THROUGHPUT OPTIMIZATION			**5. FUNDING NUMBERS**
6. AUTHOR(S) Hoang N. Tran			
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Naval Postgraduate School Monterey, CA 93943-5000			**8. PERFORMING ORGANIZATION REPORT NUMBER**
9. SPONSORING /MONITORING AGENCY NAME(S) AND ADDRESS(ES) N/A			**10. SPONSORING/MONITORING AGENCY REPORT NUMBER**
11. SUPPLEMENTARY NOTES The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government. IRB protocol number ____N/A____.			
12a. DISTRIBUTION / AVAILABILITY STATEMENT Approved for public release; distribution is unlimited			**12b. DISTRIBUTION CODE** A

13. ABSTRACT (maximum 200 words)

Speed, payload, and range are three primary interconnected variables in preliminary ship design. One design variable cannot be maximized without sacrificing the other(s). The purpose of this work is to analyze those combinations of speed, payload, and range that would give the optimal rate of cargo delivery, or throughput, in a given scenario.

A physics based mathematical model is developed to display the inter-relationship among the three primary variables. An optimization program was also developed to determine the optimal throughput for different design combinations. A sensitivity analysis was conducted to find an optimal solution that is least sensitive to changes in parameters other than the primary variables. The methodology developed in this work can be easily applied to a different ship class. The results can lead to a quick exploration of the design space in the preliminary design phase in order to isolate ranges of parameters leading to Pareto optimal sets and can be used to guide further design refinements.

14. SUBJECT TERMS Ship design, optimization, cargo throughput, sensitivity analysis			**15. NUMBER OF PAGES** 105
			16. PRICE CODE
17. SECURITY CLASSIFICATION OF REPORT Unclassified	**18. SECURITY CLASSIFICATION OF THIS PAGE** Unclassified	**19. SECURITY CLASSIFICATION OF ABSTRACT** Unclassified	**20. LIMITATION OF ABSTRACT** UU

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. 239-18

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A PRELIMINARY SHIP DESIGN MODEL FOR CARGO THROUGHPUT OPTIMIZATION

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Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN MECHANICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
June 2014

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Speed, payload, and range are three primary interconnected variables in preliminary ship design. One design variable cannot be maximized without sacrificing the other(s). The purpose of this work is to analyze those combinations of speed, payload, and range that would give the optimal rate of cargo delivery, or throughput, in a given scenario.

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LIST OF ACRONYMS AND ABBREVIATIONS

α	cargo carriage multiplier [LTs/LT]
β	weight of power factor [lbs/hp]
Δ	ship's displacement weight [LT]
Δ_{cargo}	ship's weight excluding fuel and propulsion system [LT]
Δ_{fuel}	weight of the fuel [LT]
Δ_{prop}	weight of the propulsion system [LT]
ρ	sea water density [kg/m^3]
∇	ship's displacement volume [m^3]
A	ship's wetted area [m^2]
c	admiralty coefficient [kg/m^3]
F_D	drag force [N]
Fn	volumetric Froude number
g	constant due to gravitational acceleration [m/s^2]
HSV	high speed vessel
k	refueling rate factor [hr^2/nm^2]
kts	knots
LCAC	landing craft air cushion
LT	long tons
nm	nautical miles
P	ship's payload [LT]
Ps	ship's power [Watts]
R	ship's range [nm]
S	round trip delivery distance [nm]
SFC	specific fuel consumption [lbs/hp·hr]
TP	cargo throughput [LT/hr]
V	ship's cruise speed (kts)

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ACKNOWLEDGMENTS

I would like to thank my thesis advisors for the opportunity to work on this challenging yet rewarding project. An extended debt of gratitude to Professor Papoulias, whose vision and guidance has made the duration of this project enjoyable.

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I. INTRODUCTION AND BACKGROUND

On any given day, there are approximately 110 United States Navy ships in operation that provide ocean transportation "in support of national security objectives in peace and war" [1]. As the United States Navy continue to support the needs of the world, whether due to the involvements in political conflicts or providing humanitarian supports, the demand for transporting goods to various places continue to rise. With the current economy and the downsizing in the Department of Defense's budget, a need for the optimal design of surface ships performing ocean transportation is increasingly necessary.

Various ship-design studies have been conducted in the past. Some examples of these works include: a preliminary ship design "that minimizes the economic cost of transport" [2], a parametric development to assess the effectiveness in ship design [3], and the bow and stern design of an autonomous vehicle that maximizes cargo throughput [4]. This paper differs from the above works in that it aims at providing the optimal solution based on optimal cargo delivery rate, not costs, by providing a preliminary design for a manned vessel instead of the specific designs for an autonomous vehicle, and provides actual solution instead of parameters for comparison.

This study aims at offering, first and foremost, a methodology in determining an optimal preliminary ship design for a high speed cargo ship. Secondly, it provides the optimal solution for preliminary design based on the assumptions and criteria used within this framework. The intent of this study is to provide the framework for such an optimization study and a set of results covering a wide range of design parameters.

In order to reach the objective of this study, three major milestones are required. The first milestone is a development of a physics-based mathematical model that describes the inter-relationship among the primary variables. The model needs to be sufficiently simple but rigorous to capture the fundamentals of ship design. C. B. McKesson conducted a study in which he showed that a simple model with very few parameters "can provide extremely valuable insights into the design question, and can of

course do so in a fraction of the time and energy of the more complex models" [5]. In a similar approach on a different study, he generates an equation that describes the relationship between lift to drag ratio and the volumetric Froude number [3]. A similar approach will be conducted within this study to develop the mathematical model.

Second, an optimization problem needs to be constructed to determine the optimal solution when maximized cargo throughput is desired. A. S. Wright conducted a study in which he derived an equation to calculate the cargo throughput for the landing craft air cushion (LCAC) platform [6]. This equation will be modified and used to determine the preliminary design based on the optimal throughput for high speed cargo ship in general, not just LCAC.

Third, sensitivity analyses will be conducted to provide optimal solution that has little to no impact to changes in the assumed variables.

II. DEVELOPMENT OF THE MATHEMATICAL MODEL

In order to ensure the effectiveness of the mathematical model, the data from 21 high speed vessels (HSV) was collected [7]–[25]. The HSVs are both commercial and military of various countries in the world. These data were analyzed and correlated to produce a usable relationship between the ship's displacement, cruise speed, range, and payload.

The first step was finding the trend in the relationship between ship's power and speed. More specifically, the admiralty coefficient (c) and the Froude number (Fn) were calculated. This method models after the lift to drag ratio and Froude number relationship [3]. Next, a design parameter was chosen to encompass all aspects of the ship. In this case, all parameters were expressed in terms of ship's displacement. Analysis of the output revealed the best approach in determining the effect of one parameter on the others.

A. ADMIRALTY COEFFICIENT VERSUS FROUDE NUMBER

The HSVs used to determine the correlation between c and Fn are listed in Appendix A along with the parameters used for the calculations. Fn was calculated using the volumetric method vice length as to not limit the outcome to a specific hull.

$$Fn = \frac{V}{\sqrt{g\nabla^{1/3}}} \tag{1}$$

where V is the cruise speed, g is the constant due gravitational acceleration, and ∇ is the displacement volume. The ship's power can be expressed as

$$Ps \propto F_D V \tag{2}$$

where Ps is the ship's power and F_D is the drag force. Drag force is further defined as

$$F_D \propto AV^2 \tag{3}$$

where A is the ship's wetted area. Combining Equations (2) and (3) produces an expression for power as

$$Ps \propto AV^3 \tag{4}$$

To be consistent with the Froude number approach where the length is replaced by the cubed root of the volumetric displacement, the wetted area will be expressed as the volumetric displacement to the 2/3rd power:

$$Ps \propto \nabla^{2/3} V^3 \tag{5}$$

or simply:

$$Ps = c\nabla^{2/3} V^3 \tag{6}$$

where c is defined as the admiralty coefficient. Solving for c produces

$$c = \frac{Ps}{\nabla^{2/3} V^3} \tag{7}$$

Based on Equations (1) and (7), the HSVs' parameters that are required to establish the relationship between the admiralty coefficient and the Froude number are ship's power, speed, and displacement. Appendix C is the MATLAB program generating this relationship using the calculated values of Fn and c in Appendix B. The program assumes the relationship in the form of

$$c = a + \frac{b}{Fn^m} \tag{8}$$

where m $\in$ [1, 4] with a and b are the constants established from this relationship.

Figures 1 through 4 show the resulting plots from Equation (8) for each of the m values, overlaid with the data points from the 21 chosen HSVs.

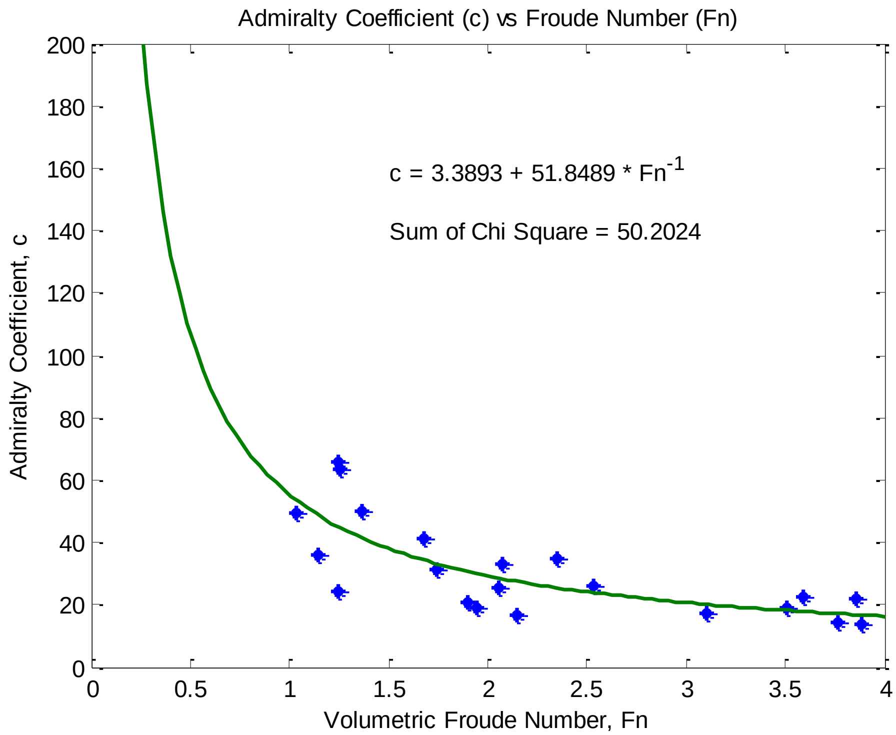

Figure 1. Admiralty Coefficient Versus Volumetric Froude Number with m = 1.

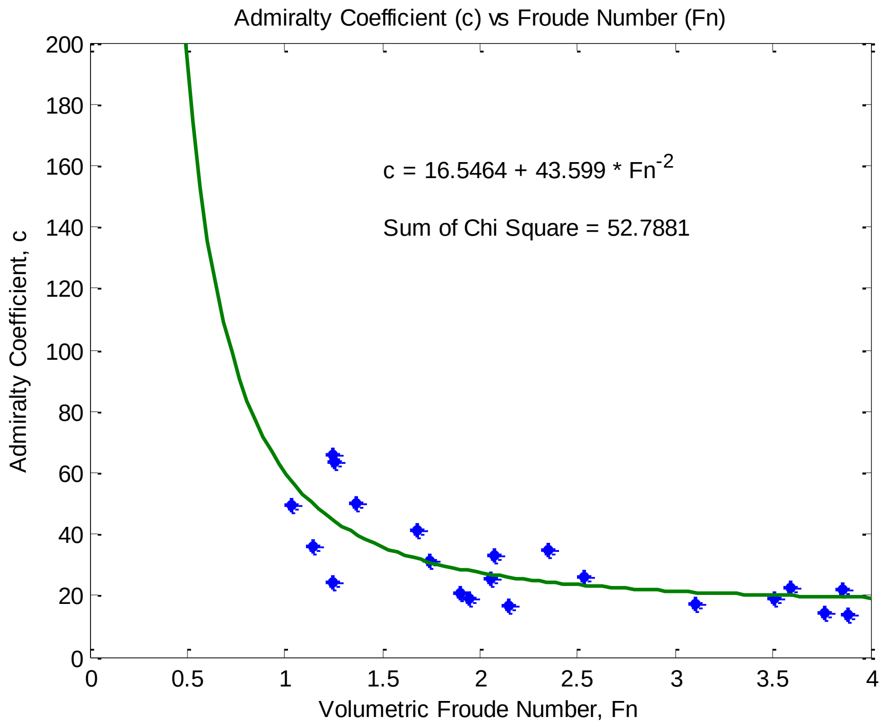

Figure 2. Admiralty Coefficient Versus Volumetric Froude Number with m = 2.

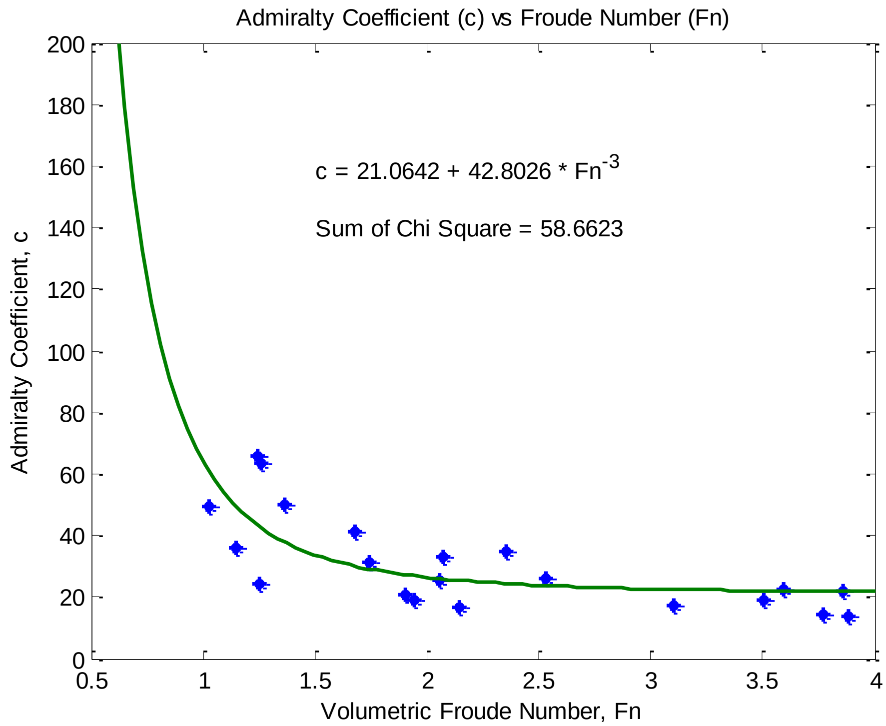

Figure 3. Admiralty Coefficient Versus Volumetric Froude Number with m = 3.

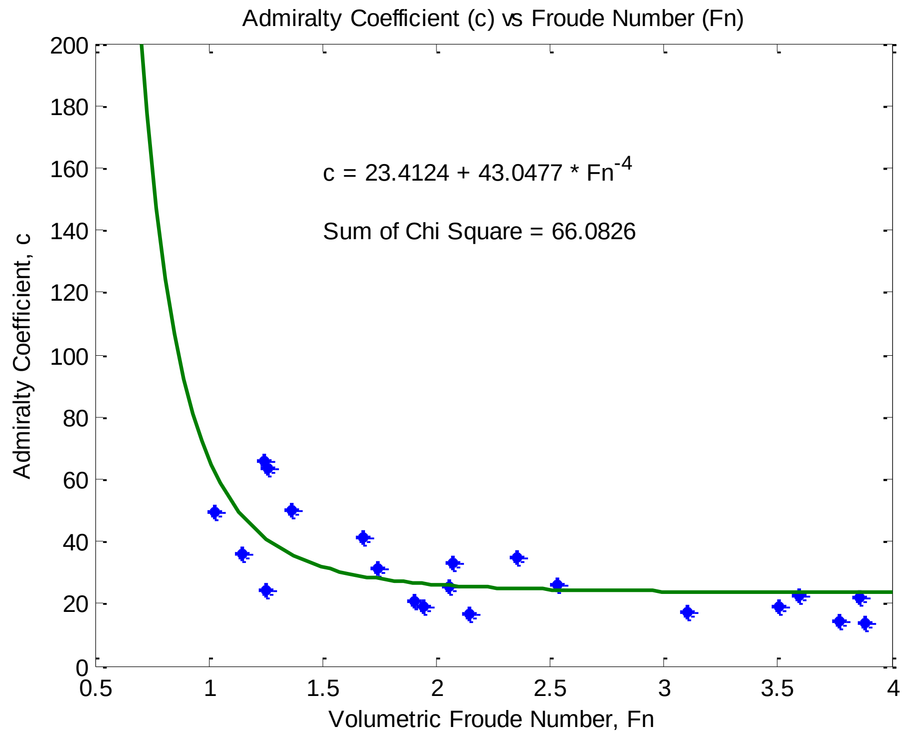

Figure 4. Admiralty Coefficient Versus Volumetric Froude Number with m = 4.

The MATLAB program also calculates the sum of chi square for each of the best fit curve based on the value of m to determine the curve that gives the least error. As seen from the four figures, m=1 has the lowest value for the sum of chi square, and therefore to be used to establish the relationship between the admiralty coefficient and the volumetric Froude number. The finalized equation becomes:

$$c = a + \frac{b}{Fn} \tag{9}$$

where a = 3.3892 and b = 51.8489.

B. THE MATHEMATICAL MODEL

Using the ship's displacement as a basis, many other parameters can be found as a function of displacement.

$$\Delta = \Delta_{prop} + \Delta_{fuel} + \Delta_{cargo} \tag{10}$$

where Δ is the total ship's displacement, Δ_{prop} is the weight of the propulsion system, Δ_{fuel} is the weight of the fuel, and Δ_{cargo} is the weight of the rest of the vessel (including the cargo).

The components of Equation (10) can be looked at individually and defined in terms of payload, power, range, and speed, the four parameters of interest.

$$\Delta_{prop} = \beta Ps \tag{11}$$

$$\Delta_{fuel} = \frac{(SFC)R}{V} Ps \tag{12}$$

$$\Delta_{cargo} = \alpha P \tag{13}$$

where β is the weight of power factor [3] [5], SFC is the specific fuel consumption, and R is the ship's range, α is the cargo carriage multiplier [3] [5], and P is the ship's payload.

Substituting Equations (11), (12), and (13) back into Equation (10) gives:

$$\Delta = \alpha P + \left(\beta + \frac{(SFC)R}{V} \right) Ps \tag{14}$$

Substituting Equations (6) for Ps and (9) for c gives:

$$\Delta = \alpha P + \left(\beta + \frac{(SFC)R}{V} \right) \left(a + \frac{b}{Fn} \right) \nabla^{2/3} V^3 \tag{15}$$

Further substituting Equation (1) for Fn and converting volumetric displacement into weight displacement results in:

$$\Delta = \alpha P + \left(\beta + \frac{(SFC)R}{V} \right) \left(a + b \frac{g^{1/2}}{V} \left(\frac{\Delta}{\rho} \right)^{1/6} \right) \left(\frac{\Delta}{\rho} \right)^{2/3} V^3 \tag{16}$$

Equation (16) is the governing equation for the relationship amongst speed, range, payload, and displacement.

C. PROGRAM OUTPUT

A MATLAB program is generated from Equation (16) using the following values for the constants:

α = 4 long tons / long ton

β = 10 lbs / hp

SFC = 0.4 lbs / hp-hr

a = 3.3892

b = 51.8489

g = 9.81 m/s^2

ρ = 1,025 kg/m^3

Constants a and b were previously generated from the admiralty coefficient equation. Constants g and ρ are universally accepted values for ship's operation in sea water on earth. The specific fuel consumption assigned value is a generally accepted value for ship machinery. The values of α and β were estimated and taken from McKesson's works [3]. Since these are neither universal constants nor generally accepted values, analyses on the effects of their variation are needed and will be conducted in Section IV.

If ship's speed, range, and payload are known, a simple iteration can be accomplished to produce the required displacement. For example, for

V = 45 knots (kts)

P = 400 long tons (LT)

R = 3,512 nautical miles (nm)

The MATLAB program in Appendix D would give a displacement of 4,000 LT. On the other hand, if only the displacement is known, how are speed, range, and payload related to one another?

1. Speed, Range, and Payload Relationship

Appendix E contains the MATLAB program that determines the speed, range, and payload limits for a given displacement. In this particular case, a displacement of 4,000 LT is chosen. Figures 5 through 7 display three different variations in depicting this relationship.

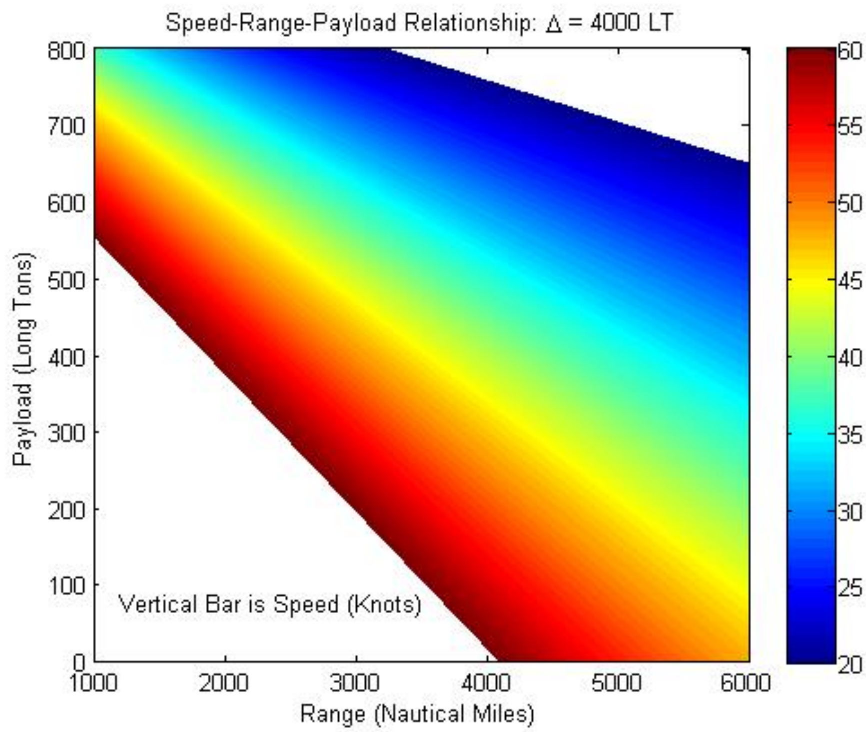

Figure 5. Payload Versus Range in Variable Speed.

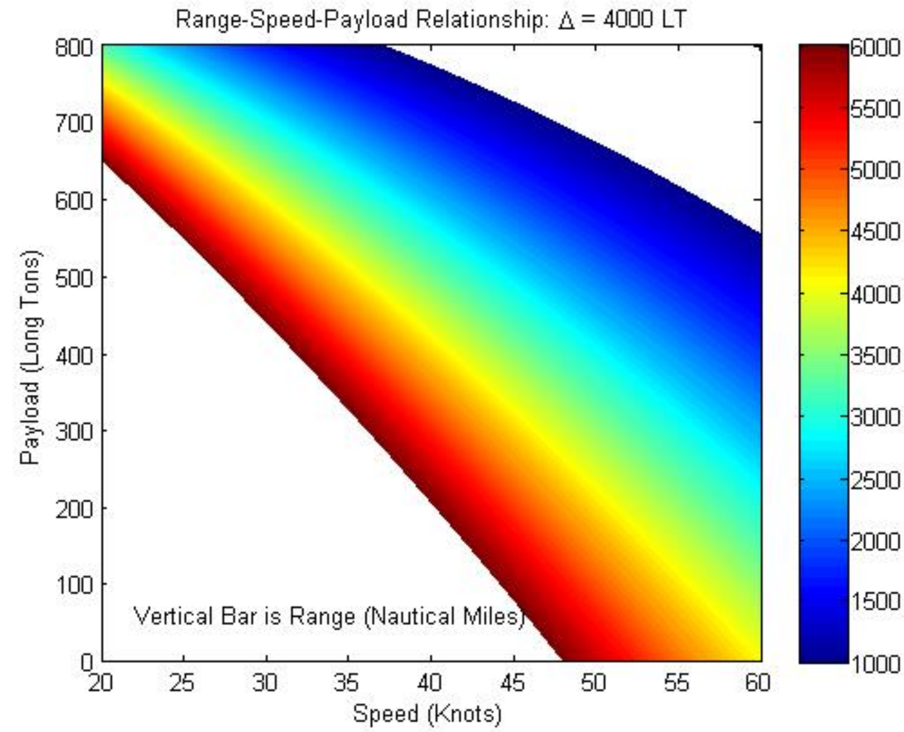

Figure 6. Payload Versus Speed in Variable Range.

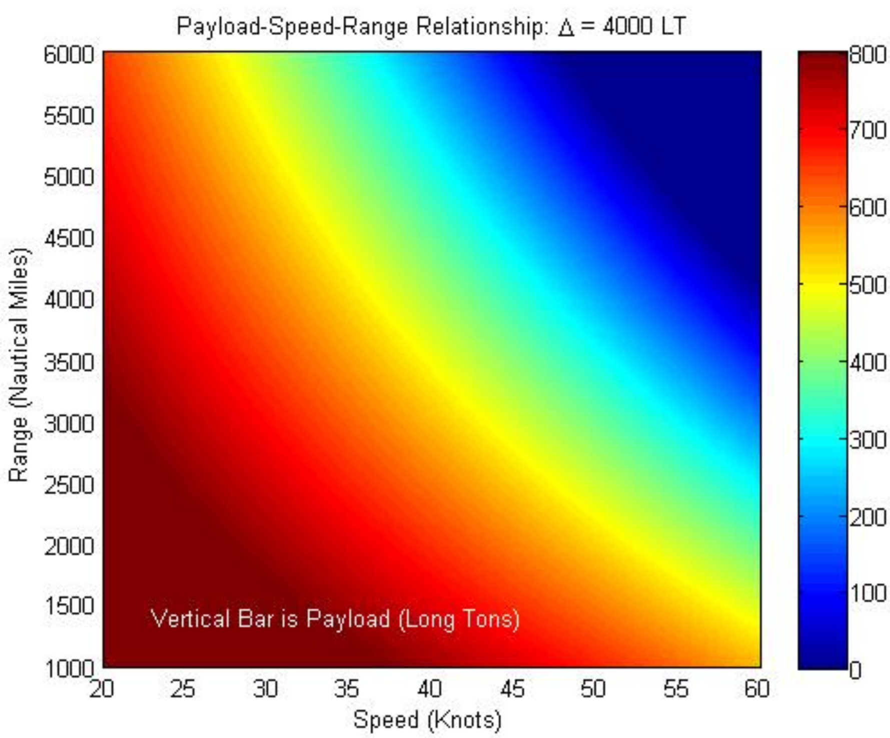

Figure 7. Range Versus Speed in Variable Payload.

Of the three graphs, only Figure 5 results in a linear relationship. Therefore, further exploration of this linear relationship is necessary.

2. Usable Output

Appendix G provides the MATLAB program that further breaks down the payload versus range relationship in an incremental speed variation. Figure 8 is the result for this approach.

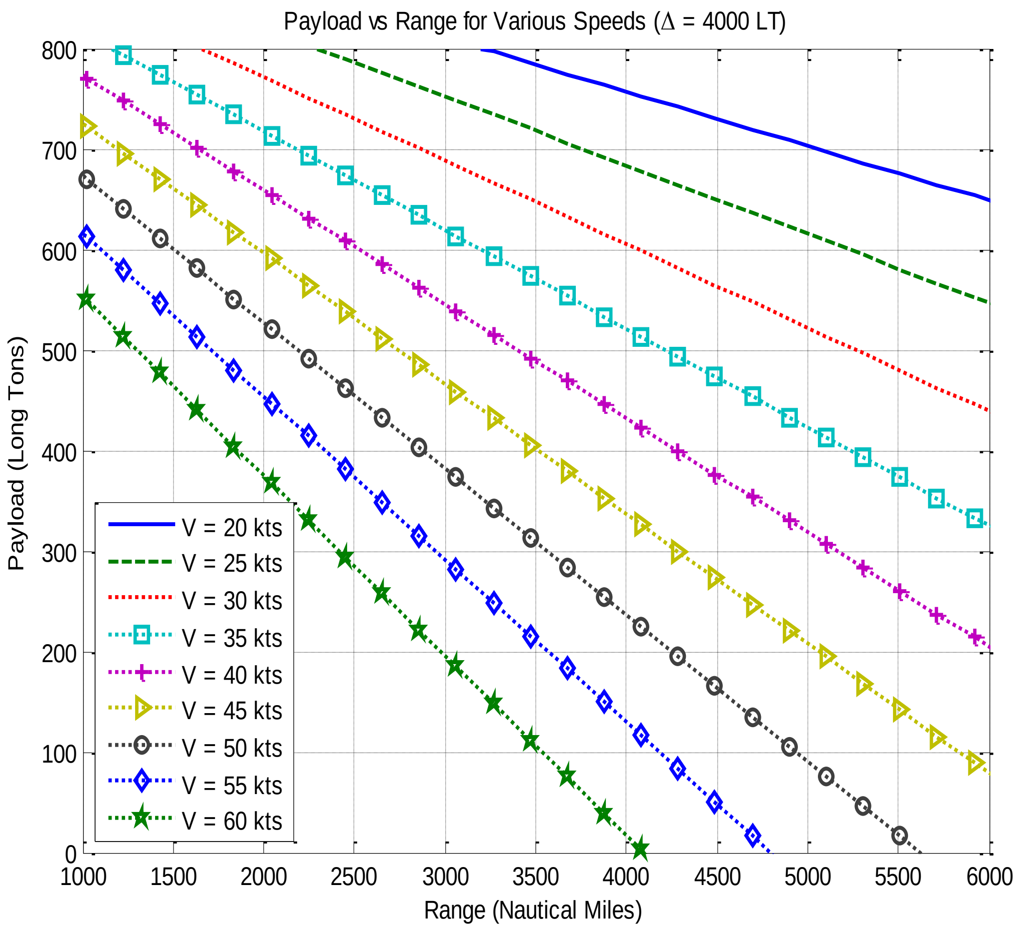

Figure 8. Payload vs. Range for Discrete Values of Speed.

From the first order curves displayed in Figure 8, an equation for payload can be expressed as:

$$P = A(V) + B(V)R \tag{17}$$

where A(V) is the y-intercept and B(V) is the slope of the secant, both as function of speed. Appendix G also provides the values of A and B as functions of speed, for a given displacement. Table 1 gives the values in tabular form and Figure 9 gives the graphical results.

Speed (kts)	A	B
20	973.0	-0.0541
25	957.2	-0.0684
30	937.6	-0.0832
35	914.1	-0.0982
40	886.4	-0.1136
45	854.4	-0.1294
50	818.1	-0.1455
55	777.3	-0.1620
60	731.9	-0.1788

Table 1. Constants A and B for Displacement of 4,000 LT.

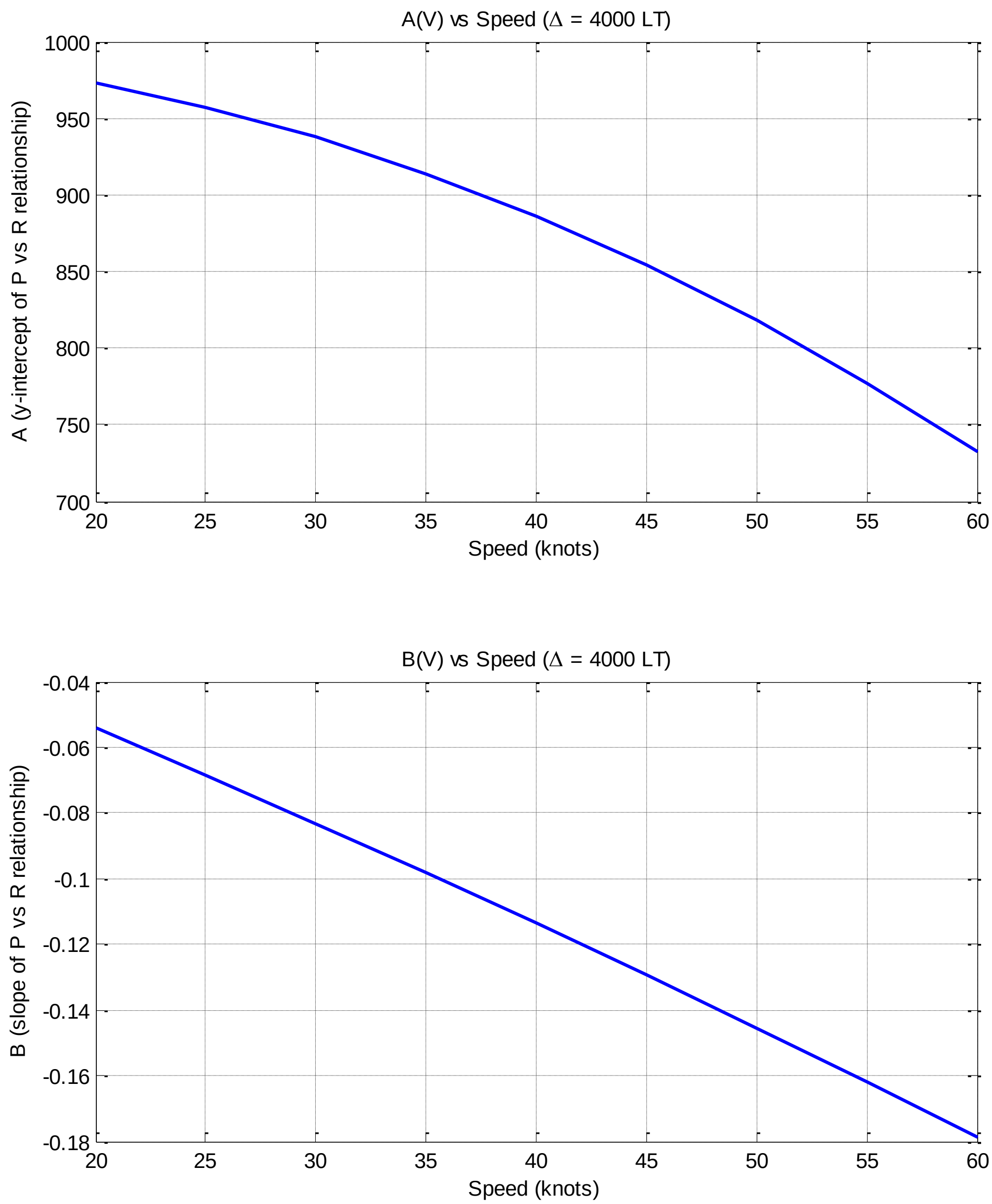

Figure 9. Constants A and B as Functions of Speed.

3. The Tabular Form

Instead of graphical presentation of payload vs. range in variable speed for a given displacement, another approach is using the tabular format. Appendix F contains

the MATLAB program that generates Table 2 and Figures 10 and 11. The derivation of these constants comes from re-arranging Equation (16) to into the form of Equation (17), which produces the following relationship:

$$A(V) = a_1 + a_2 V^2 + a_3 V^3 \tag{18}$$

$$B(V) = b_1 V + b_2 V^2 \tag{19}$$

Displacement (LT)	a_1	a_2	a_3	b_1	b_2
1000	250	-0.0202	-0.000069	-0.00081	-0.00000275
1500	375	-0.0283	-0.000090	-0.00113	-0.00000360
2000	500	-0.0360	-0.000109	-0.00144	-0.00000436
2500	625	-0.0433	-0.000126	-0.00173	-0.00000506
3000	750	-0.0504	-0.000143	-0.00202	-0.00000571
3500	875	-0.0573	-0.000158	-0.00229	-0.00000633
4000	1000	-0.0641	-0.000173	-0.00256	-0.00000692
4500	1125	-0.0707	-0.000187	-0.00283	-0.00000748
5000	1250	-0.0772	-0.000201	-0.00309	-0.00000803
5500	1375	-0.0836	-0.000214	-0.00334	-0.00000856
6000	1500	-0.0899	-0.000227	-0.00359	-0.00000907

Table 2. Constants a_1, a_2, a_3, b_1, and b_2 as Functions of Displacement.

With these results, the design parameters of the vessel can easily be determined. A simple four-step process will guide to the solution:

1. Choose a design displacement.
2. Obtain a_1, a_2, a_3, b_1, and b_2 by either using Table 2 or Figures 10 and 11.
3. Substituting these values into Equations (18) and (19) to get A and B, with a desired cruise speed.
4. Substitute A and B into Equation (17) to get the relationship between payload and range.

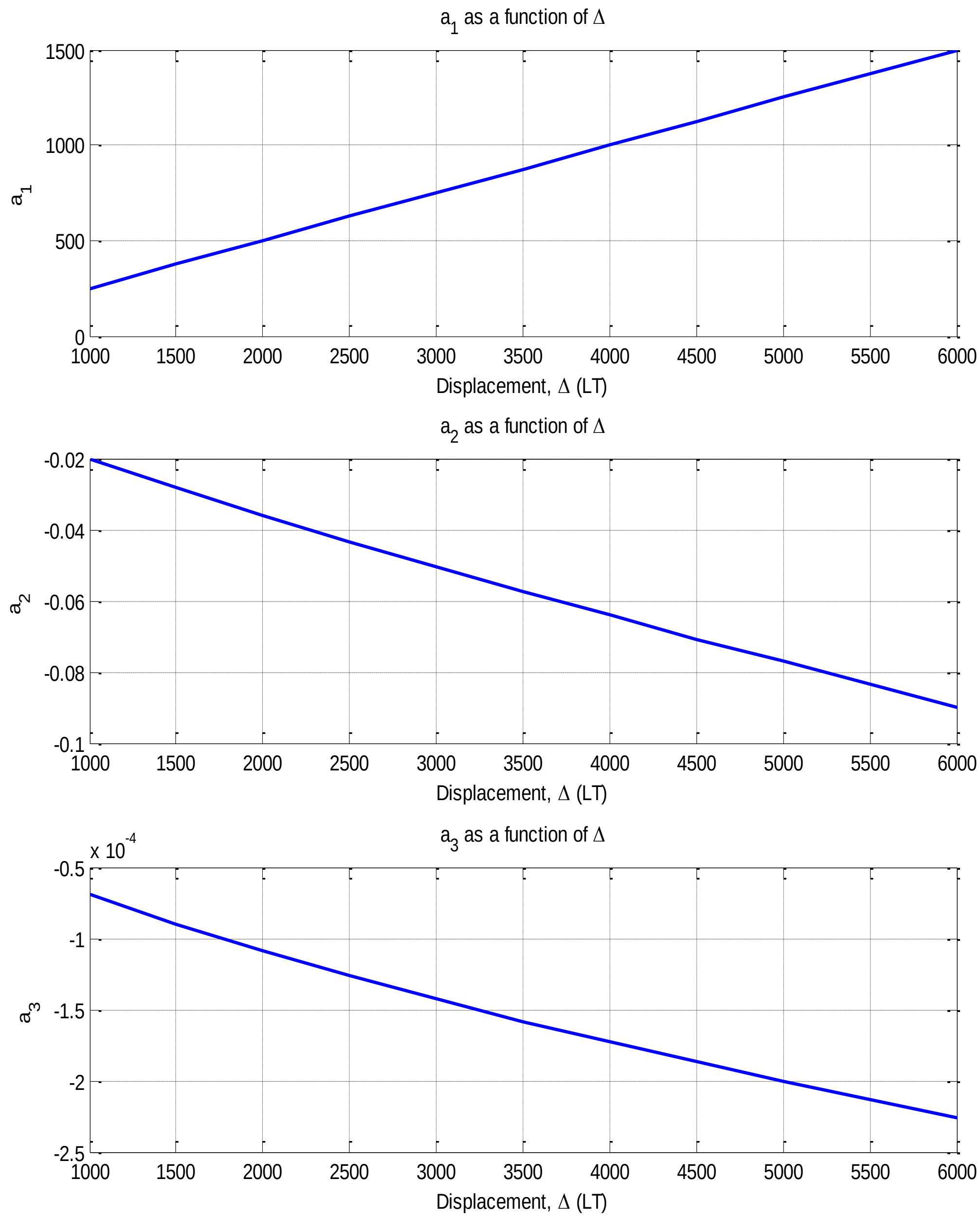

Figure 10. Constants a_1, a_2, and a_3 as Functions of Displacement.

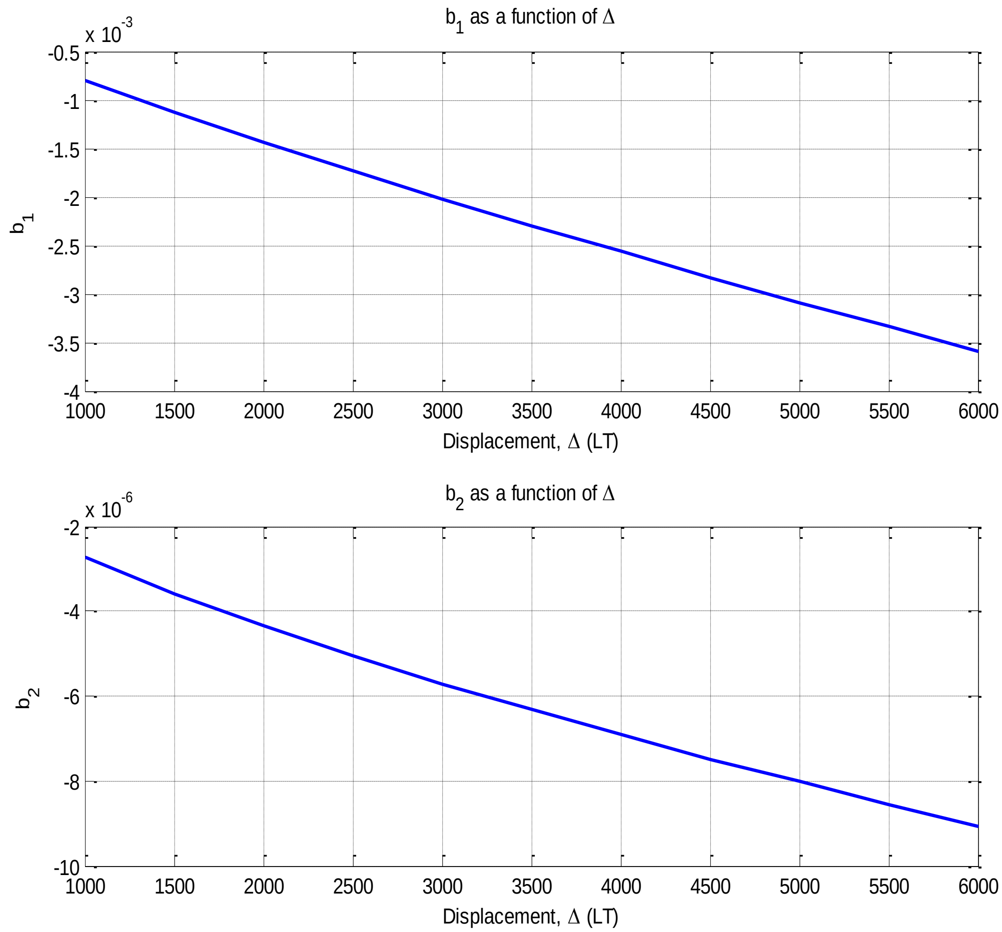

Figure 11. Constants b_1, b_2 as Functions of Displacement.

These are the results of a physics based mathematical model that is simple yet rigorous. The next step is determining the optimal design solution.

III. THROUGHPUT OPTIMIZATION

The developed mathematical model provides an infinite number of combinations for the preliminary ship design. The interest of this study is to optimize the cargo throughput. Therefore, a generation of an optimization problem is required to accomplish this step. The objective function and the constraint(s) to this optimization problem need to be defined.

A. THE OPTIMIZATION PROBLEM

1. The Objective Function

Since the goal is to maximize the cargo throughput, the objective function needs to be an equation or expression for throughput. A. S. Wright derived the following equation to calculate the cargo throughput for the LCAC: [6]

$$Z_i = \frac{x_1}{\dfrac{2x_2}{x_3} + c_1 + c_2 + \dfrac{x_1}{c_1} + \dfrac{\dfrac{x_4}{c_4}}{\dfrac{x_4}{c_5}\dfrac{x_3}{x_2}}} \tag{20}$$

where: $Z_i = cargo\ throughput\ (tons/min)$

$x_1 = cargo\ (tons)$

$x_2 = distance\ (nm)$

$x_3 = speed\ (nm/min)$

$x_4 = fuel\ loaded\ (gal)$

$c_1 = receive\ delay\ (min)$

$c_2 = send\ delay\ (min)$

$c_3 = load + unload\ rate\ (tons/min)$

$c_4 = fueling\ rate\ (gal/min)$

$c_5 = fuel\ consumption\ rate\ (gal/min)$

Wright went on to simplify this equation into the following based on a 30 minutes send and receive delay, 0.2 tons/min loading and unloading rate and a known refueling rate and fuel consumption rate ratio [6]:

$$Z_i = \frac{x_1}{\frac{2.016x_2}{x_3} + 30 + 5x_1} \tag{21}$$

It is necessary to modify Wright's equation for two reasons: 1) Wright's study is specific to LCAC where he knows the fuel consumption rate and 2) the amount of fuel required for each refueling is constant due to a non-varying ship's range.

The modified throughput equation, using the variables in this study, becomes:

$$TP = \frac{P}{\frac{S}{V} + 0.5 + \frac{P}{12} + kSV} \tag{22}$$

where $TP = cargo\ throughput\ \left(\frac{LT}{hr}\right)$

$P\ = payload\ (LT)$

$S\ = round\ trip\ delivery\ distance\ (nm)$

$V\ = ship's\ cruise\ speed\ (kts)$

$k\ = f(R, V, refuel\ rate)\ in\ (\frac{hr^2}{nm^2})$

Note that the assumptions used by Wright in regard to send and receive delay and loading and unloading rate are used in Equation (22). Instead of 30 minutes, 0.5 hours is used; instead of 0.2 tons/min, 1/12 tons/hrs are used.

Equation (22) introduced the variable k, which is a function of the refueling rate, ship's range (fuel capacity), and ship's speed. It is calculated by using Equation (12) to determine the amount of fuel based on a given speed and range, then applied the assumed refueling rate of 120,000 gal/hr (or 2,000 gal/min) to find the refueling time. The particular k is the ratio of time to the product of particular speed and range. The generic k

used in Equation (22) is the average over different combinations of speed and range within the developed mathematical model. Table 3 provides different values of k based on refueling rate and design displacement.

k		1,000 gal/min	2,000 gal/min	4,000 gal/min	8,000 gal/min
1,000 LT		1.01E-04	5.03E-05	2.52E-05	1.26E-05
2,000 LT		1.78E-04	8.87E-05	4.44E-05	2.22E-05
3,000 LT		2.47E-04	1.24E-04	6.18E-05	3.09E-05
4,000 LT		3.13E-04	1.57E-04	7.83E-05	3.91E-05
5,000 LT		3.76E-04	1.88E-04	9.40E-05	4.70E-05
6,000 LT		4.37E-04	2.18E-04	1.09E-04	5.46E-05

Table 3. Refueling Rate Factor Based on Rate of Refueling and Design Displacement.

2. The Constraint Function

The constraint is the mathematical model developed in the previous section, Equation (16), since the values of the speed, range, and payload must be within the feasible region of their inter-relationship.

3. Setting up the optimization problem

By convention, all computer programs solve the optimization problems by minimizing the objective function subjecting to the given constraint(s) [26]. In this study, the interest is in maximizing the throughput. Therefore, Equation (22) needs to be multiplied by a (-1) since maximizing a function is the same as minimizing its negative.

Also by convention that constraint function(s) have zero on the right hand side of the equation and everything else on the left [26], so Equation (16) needs to be re-arranged into the correct format.

The optimization problem becomes:

$$\text{minimize} \quad \frac{-P}{\frac{S}{V}+0.5+\frac{P}{12}+kSV}$$

$$\text{subject to} \quad \alpha P+\left(\beta+\frac{(SFC)R}{V}\right)\left(a+b\frac{g^{1/2}}{V}\left(\frac{\Delta}{\rho}\right)^{1/6}\right)\left(\frac{\Delta}{\rho}\right)^{2/3}V^{3}-\Delta\leq 0$$

4. Convexity of the functions

If both the objective and constraint functions are convex then there exists only one optimal solution [26]. Unfortunately, the objective function is not convex. Therefore, there exists more than one local optimal solution. Depends on the initial guess for the solution, the computing program (in this case MATLAB) would drive the solution to a particular point.

One way of handling this issue is to analyze the objective function to choose the initial guess that makes sense in terms of optimality. In regard to speed, range, and payload, to maximize the amount of payload per unit time means maximizing payload, maximizing speed, and minimizing range. Therefore, the initial guess for the optimization problem is to be the maximum of the payload and speed along with the minimum range within the constraints of this study. Appendix H contains the MATLAB codes that solved the above optimization problem.

B. PROGRAM OUTPUTS

Due to the nature of this optimization problem, the MATLAB program always outputs the lowest value in the range interval as the optimal value for ship's range. In order to see how speed or payload changes with respect to range, different values of range are fixed to produce the optimal associated speed and payload. Appendix I contains the MATLAB program that produced various results based on the developed optimization problem.

Figure 12 displays the curves showing the relationship between the optimal speed versus range for different refueling rates (or k) while holding the design displacement constant. Recall that variable k was introduced in the objective function. This k is a

function of range, speed, and refueling rate, which is not one of the three primary variables of interest. Therefore, different result is obtained for different value of k. The upper most curve displays the values of ship whose range is greater than the round-trip delivery distance, hence no refueling required (k = 0). The subsequent curves associated with the particular refueling rate. As the refueling rate increases, its optimal curve approaches the one without refueling required.

Figure 13 displays same set of results graphing optimal payload versus speed. In this plot, the no refueling curve is at the bottom with other curves lowered as the refueling rate increases. This is the nature of optimization problem. In this particular scenario speed plays a more important factor than payload in terms of optimizing the cargo throughput.

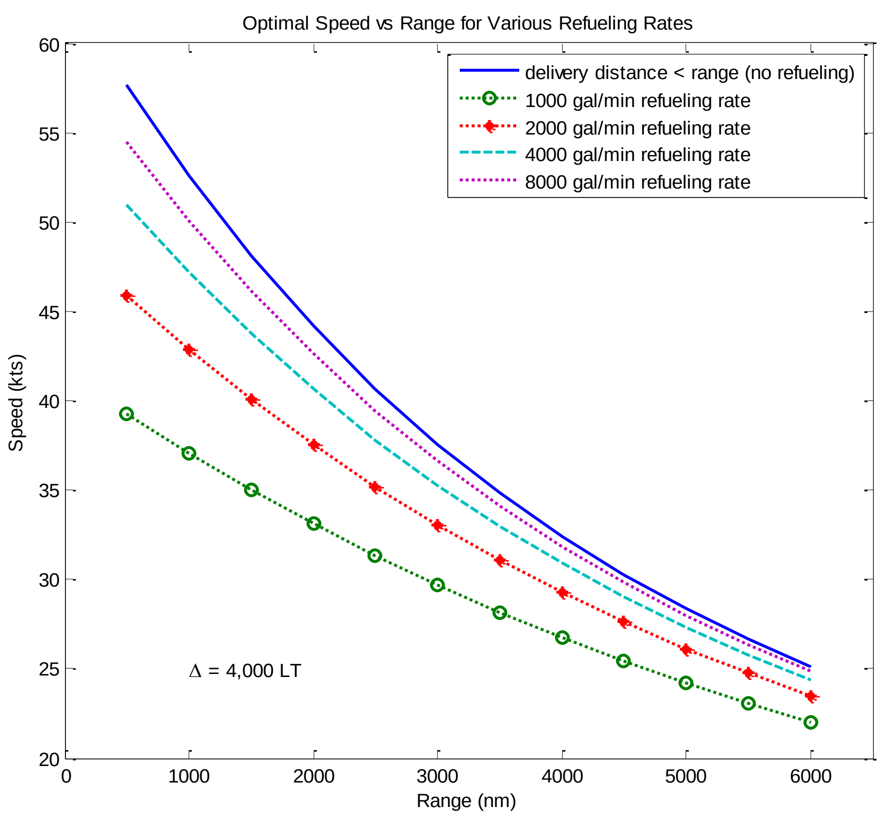

Figure 12. Optimal Speed Versus Range for Various Refueling Rates (Δ = 4,000 LT).

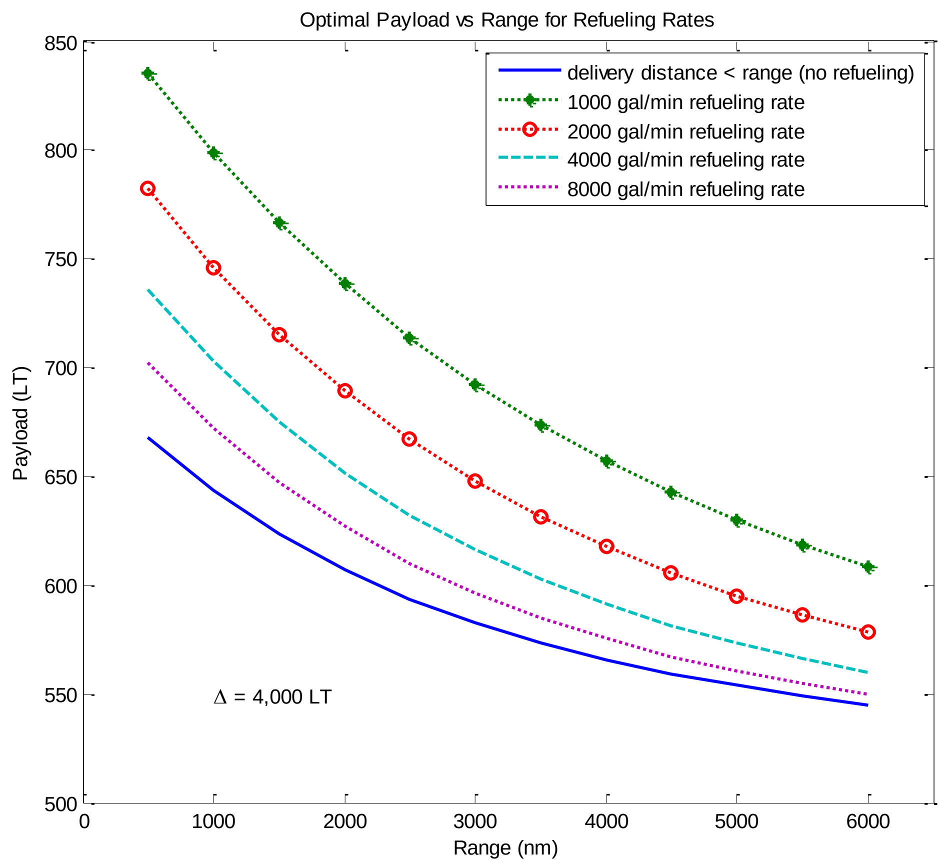

Figure 13. Optimal Payload Versus Range for Various Refueling Rates (Δ = 4,000 LT).

Of note, the results from this optimization program assumed that refueling can be done at any time. In a realistic operational environment, this might not be so. Therefore, based on the constraint of refueling capability, the ship's range might have a much higher lower limit than 500 nm set in this study. For example, if refueling cannot be accomplished until the ship had traveled for at least 2,800 nm then the optimal range should be around 3,000 nm with its associated optimal speed of around 33 knots at 2,000 gal/min refueling rate.

Once the minimum range is known, the associated optimal speed can be determined from Figure 12, the optimal payload from Figure 13, and cargo throughput are obtained based on the design displacement for the ship.

IV. SENSITIVITY ANALYSIS

Thus far the optimal solution for maximizing cargo throughput were based on the estimated values for α (cargo carriage multiplier), β (weight of power factor), and k (the refueling rate factor). It is paramount to provide a preliminary design such that the variations in these values have minimal impact on the output solution.

A. SENSITIVITY OF THE CARGO CARRIAGE MULTIPLIER

The developed optimization program was re-run using α equals to 2 instead of 4, meaning that the weight of the ship, not counting fuel and propulsion weights, is two times that of the amount of payload. Holding everything else constant except the three primary variables (displacement = 4,000 LT and refueling rate at 2,000 gal/min), it was found that the optimal range and speed for both instances are the same. In fact, for all α values, the optimal range and speed do not change, only the payload. For example, for α = 2, its optimal payload value is exactly twice that of the optimal payload value for α = 4. In other word, the following relationship is true:

$$\alpha_i P_i = constant \tag{23}$$

or

$$\alpha_1 P_1 = \alpha_o P_o \tag{24}$$

Since the α value used in the program equals to 4, the new optimal payload based on varying α value can be determined by:

$$P_i = \frac{4P_o}{\alpha_i} \tag{25}$$

where

$P_o =$ the optimal payload value based on α = 4.

$\alpha_i =$ actual cargo carriage multiplier.

$P_i =$ actual optimal payload based on actual cargo carriage multiplier.

Appendix J contains the MATLAB program that conducts the sensitivity analysis on the cargo carriage multiplier and produced the results in Figure 14 for two different α values.

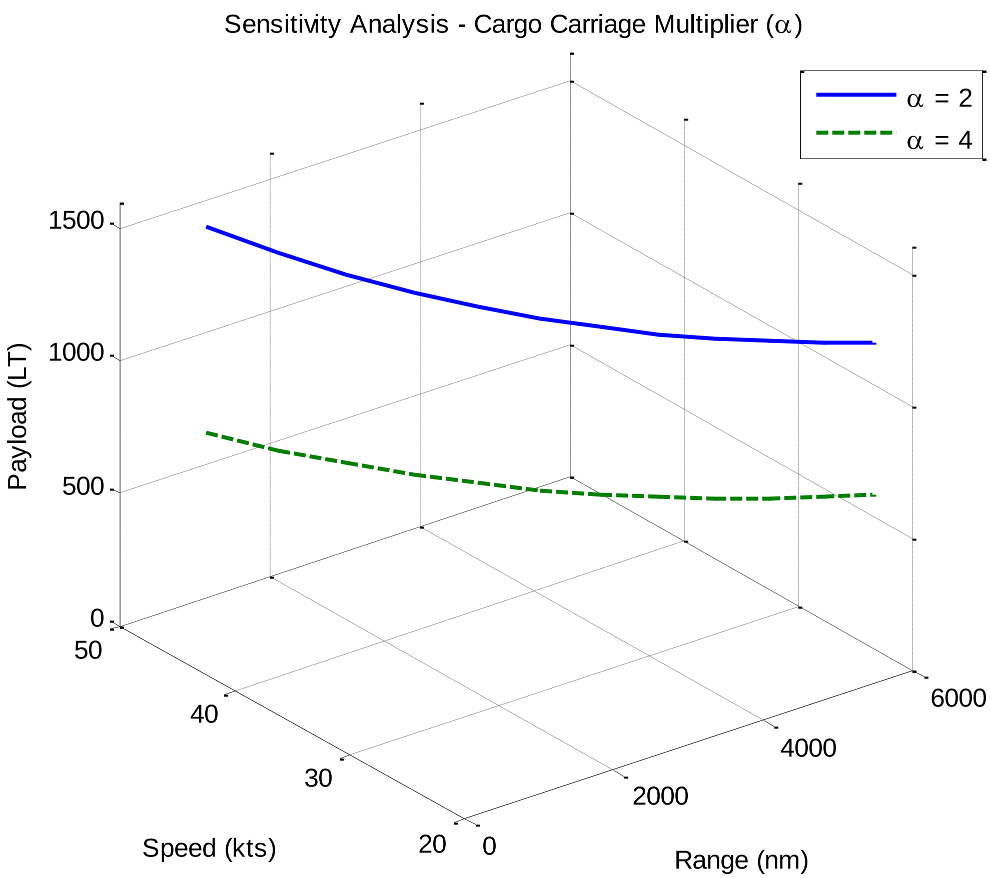

Figure 14. Sensitivity Analysis of the Cargo Carriage Multiplier.

B. SENSITIVITY OF THE WEIGHT OF POWER FACTOR

As performed for the cargo carriage multiplier, the same conditions were held constants (displacement of 4,000 LT and refueling rate of 2,000 gal/min), except the three primary variables, while varying β. Three different plots were generated for trend analysis, optimal speed versus range, optimal payload versus range, and optimal throughput versus range. Figures 15–17 displayed the results of these plots respectively.

Appendix K contains the generic MATLAB codes to generate different plots within the weight of power factor sensitivity analyses section.

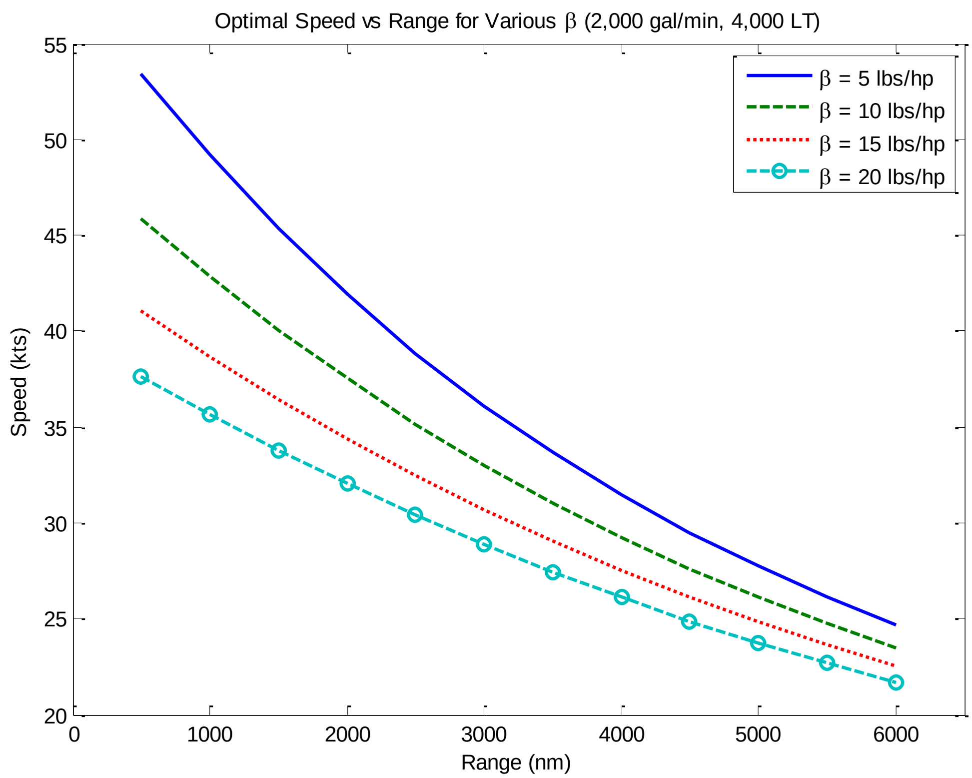

Figure 15. Sensitivity Analysis (Speed vs Range) of the Weight of Power Factor Based on 2,000 gal/min Refueling Rate and 4,000 LT Design Displacement.

Both the speed versus range (Figure 15) and throughput versus range (Figure 17) showed the sensitivity of β decreases with an increase in range (or decrease in speed and throughput). With the maximum range of 6,000 nautical miles assigned within the framework of this study, the variation in speed and throughput are not small enough to consider β as insensitive.

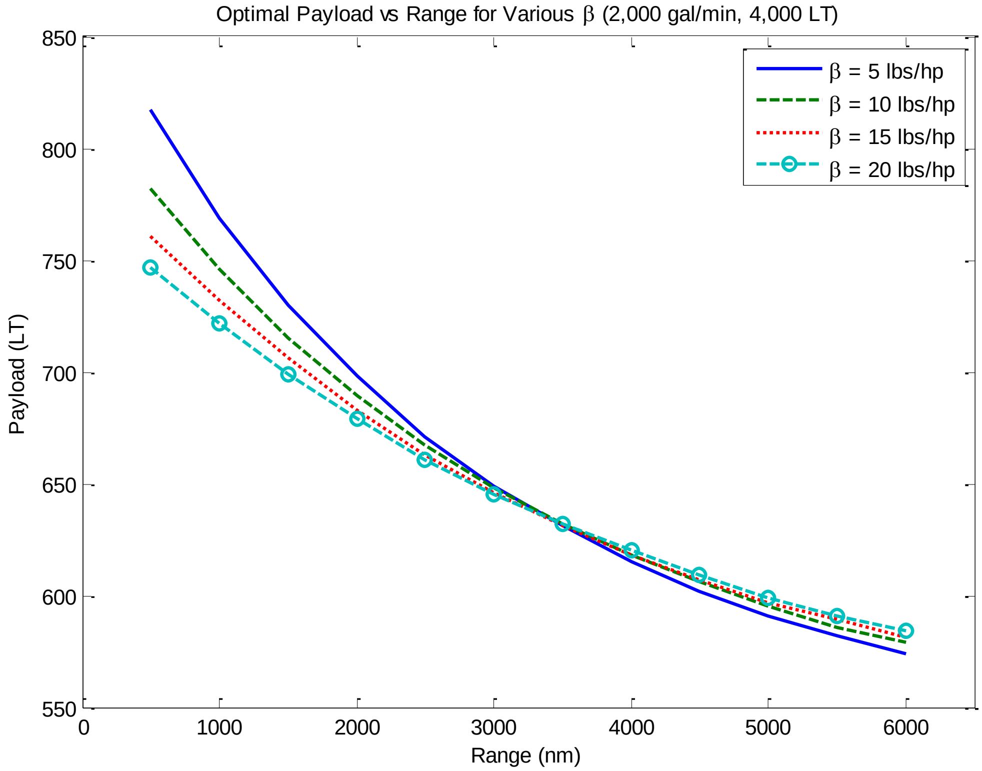

Figure 16. Sensitivity Analysis (Payload vs Range) of the Weight of Power Factor Based on 2,000 gal/min Refueling Rate and 4,000 LT Design Displacement.

However, in Figure 16, there is a definite optimal range (and associated payload) at which the variation in β has no effects. In this case, the optimal range is 3,500 nautical miles with an optimal payload of 631 LT. With these values, along with the pre-conditioned displacement of 4,000 LT, the optimal speed was calculated from Equation (16) to be 31.05 knots.

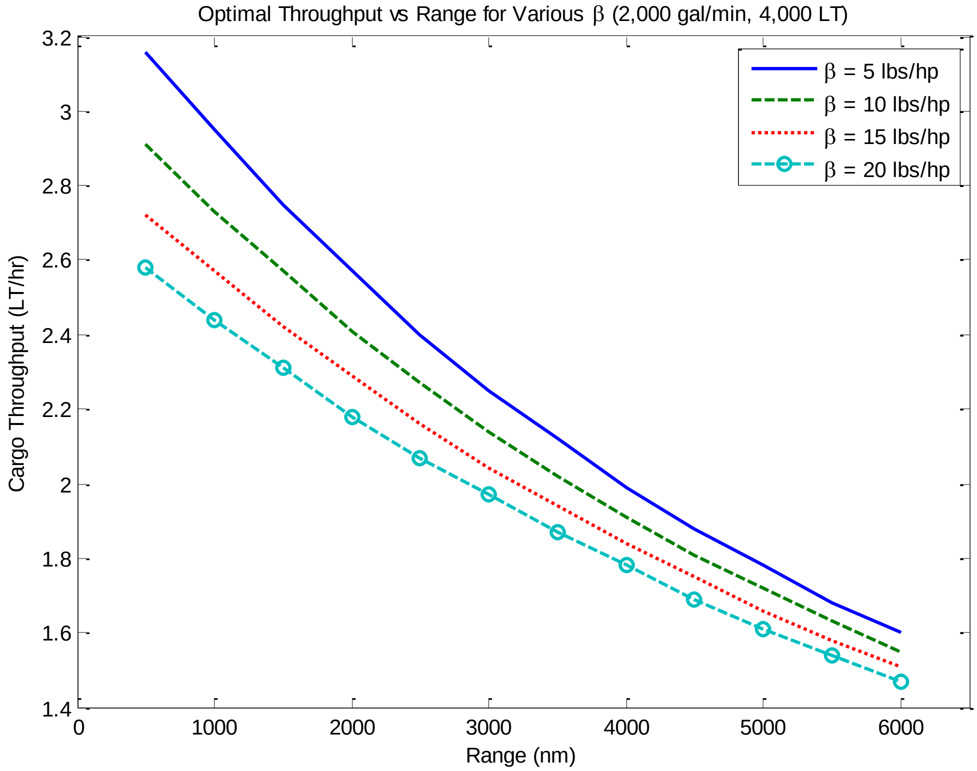

Figure 17. Sensitivity Analysis (Throughput vs Range) of the Weight of Power Factor Based 2,000 gal/min Refueling Rate and 4,000 LT Design Displacement.

To ensure this trend is reliable for substantial conclusion, the process was re-run with different values of design displacement. Figures 18 and 19 showed the optimal payload versus range plots for 3,000 LT and 5,000 LT displacements respectively, while still holding the refueling rate at 2,000 gallons per minute.

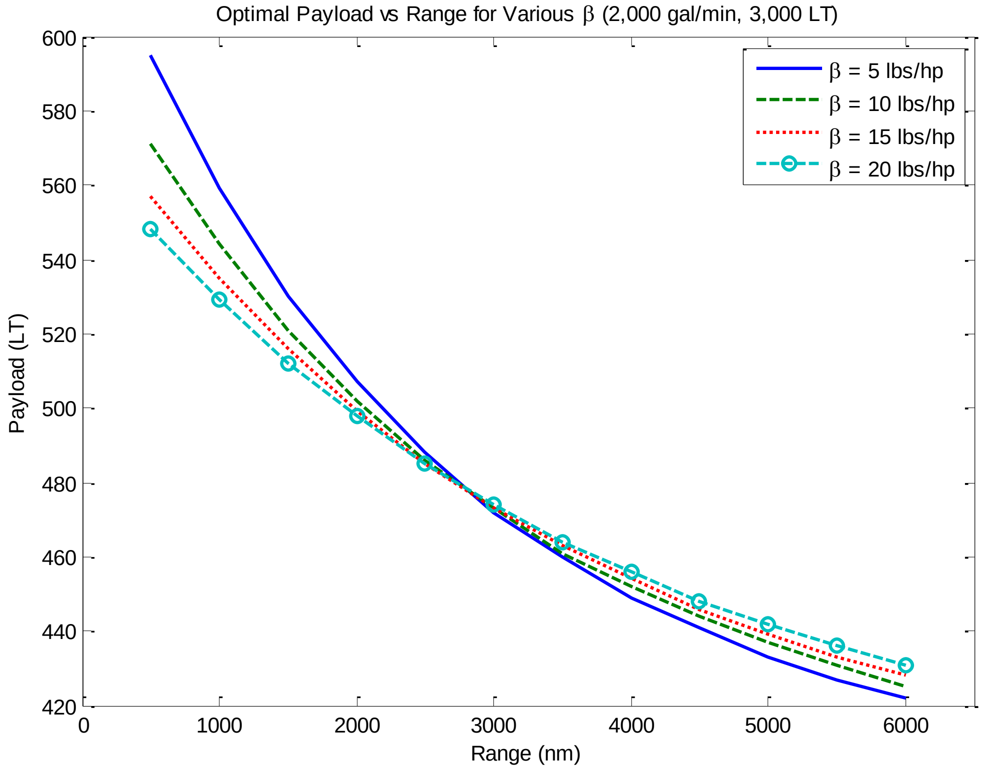

Figure 18. Sensitivity Analysis (Payload vs Range) of the Weight of Power Factor Based on 2,000 gal/min Refueling Rate and 3,000 LT Design Displacement.

Figures 18 and 19 proved the consistency in the sensitivity of the weight of power factor. For a design displacement of 3,000 LT the optimal range that is insensitive to the change in β is 2,800 nautical miles, and 4,000 nautical miles for a 5,000 LT design displacement. This process was repeated for different values of displacements from 1,000 LT to 6,000 LT with a 1,000 LT interval. The results were recorded in Table 4.

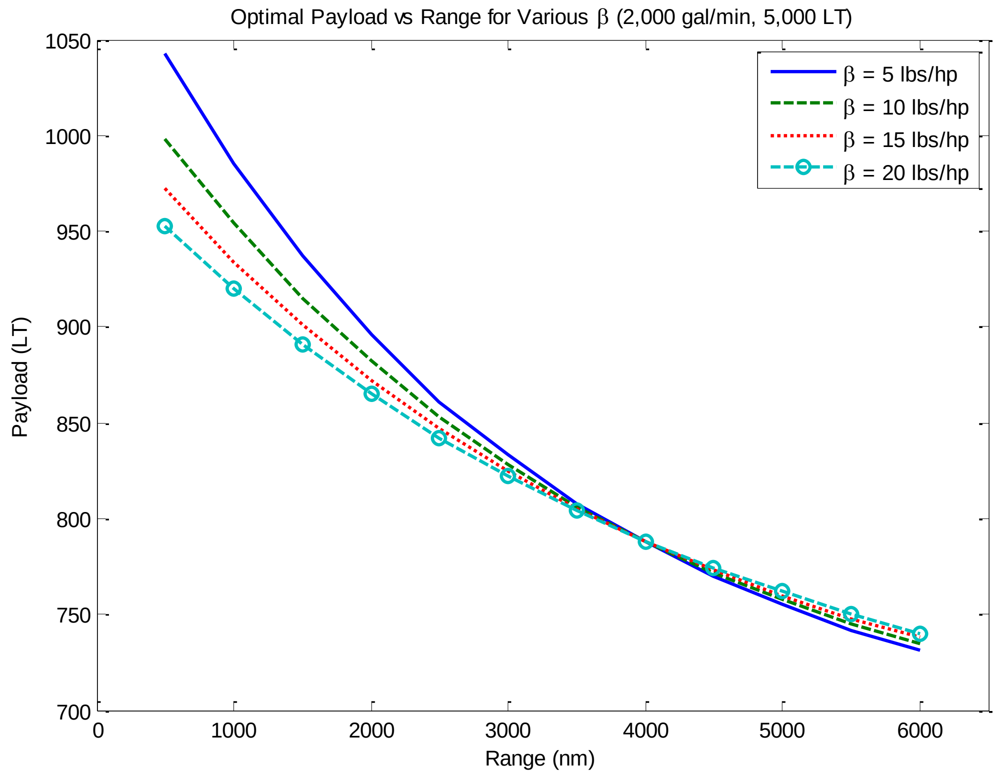

Figure 19. Sensitivity Analysis (Payload vs Range) of the Weight of Power Factor Based on 2,000 gal/min Refueling Rate and 5,000 LT Design Displacement.

Displacement (LT)	Speed (kts)	Range (nm)	Payload (LT)
1000	40.67	1337	162
2000	36.56	2168	320
3000	33.77	2801	478
4000	31.05	3501	631
5000	29.37	4002	788
6000	27.87	4497	943

Table 4. Optimal Preliminary Design Based on Sensitivity of β

C. SENSITIVITY OF THE REFUELING RATE

As previously shown that different refueling rate would produce different optimal solution to the optimization problem. Furthermore, a single refueling rate of 2,000 gallons per minute was used in the sensitivity analysis for the weight of power factor. So, what impact would a change in the refueling rate has on the optimal solutions presented in part C of this section?

The 2,000 gallons per minute refueling rate was the average value determined by a typical refueling at sea based on the amount of fuel and the total time elapsed from beginning to end. The 1,000 gallons per minute refueling rate (half as fast) and the 4,000 gallons per minute refueling rate (twice as fast) will be used in comparison with the 2,000 gallons per minute refueling rate for comparison. Appendix L provides the MATLAB codes that produced the results in Figure 20 based on a fixed displacement of 4,000 LT.

Two sets of curves are presented in Figure 20. The solid lines represent the 1,000 gallons per minute refueling rate while the dashed lines represent the 4,000 gallons per minute refueling rate. The vertical dotted line represents the location of the optimal range that was insensitive to β for a 2,000 gal/min refueling rate. At this range (3,500 nm) the maximum difference for different β values are 9 and 11 LT. This is approximately 0.25 percent of the total ship's displacement and less than two percent of the total payload. These are negligible values when viewing from a grand scheme in the preliminary design. In addition, the actual variation in the refueling rate would be much closer to the estimated 2,000 gallons per minute vice the analysis values used for half and twice the rate.

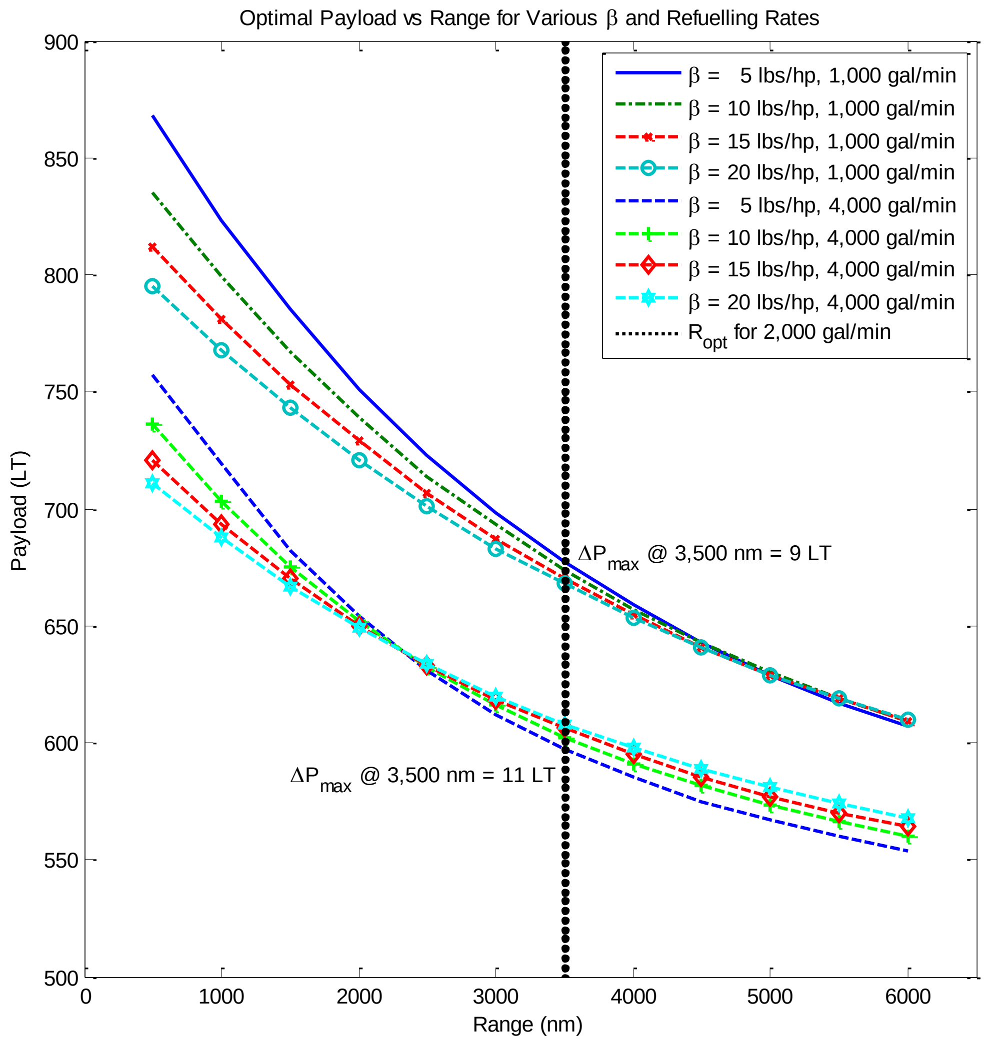

Figure 20. Sensitivity Analysis (Payload vs Range) of the Effect of the Refueling Rate on the Weight of Power Factor for a Design Displacement at 4,000 LT.

Based on these results, the actual variation of the refueling rate from the estimated 2,000 gallons per minute does not have considerable effect on the weight of power factor analysis, and therefore, has minimal impact on the optimal solution as a whole.

As technology and processes improved, the refueling rate might increase to a value higher than that of 2,000 gallons per minute. The results showed that the least

sensitive value of ship’s range shifted to the left (less than 3,500 nm) as the refueling rate increased. Therefore, using an optimal range of 3,500 nm is more conservative with respect to higher refueling rate.

The same analyses were conducted for the rest of the interested design displacement values and found that the optimal solutions based on the 2,000 gallons per minute refueling rate were acceptable in regard to both conservatism and minimal impacts due to a change in β.

D. SENSITIVITY ANALYSIS SUMMARY

The cargo carriage multiplier has no effects on the optimal range and optimal speed, with the optimal payload as the multiple of the inverse of the ratio of the cargo carriage multiplier. No changes required in regard to the optimization problem results. If an actual cargo carriage multiplier is known, simply calculate the new optimal payload using Equation (25).

The weight of power factor produced an interval of optimal design range at which minimal changes in the optimal payloads were observed and a specific optimal design range at which the optimal payload was insensitive to the change in the weight of power factor. Therefore, though the optimization program optimizes the problem towards the lowest value in the design range interval, the design range had to be limited to a value that its sensitivity to the change in the weight of power factor was limited.

As the refueling rate goes up, the design range that is insensitive to the change in the weight of power factor moved to the left, providing higher optimal values from the optimization problem. The 2,000 gallons per minute refueling rate was chosen due to its conservatism toward higher refueling rate with minimal impact in range and payload with regard to the lower refueling rate.

V. SIMULATION RESULTS

The results of the sensitivity analysis showed that the values in Table 4 hold as the optimal solutions that were least sensitive to the changes in α, β, and k. Based on these, the optimization program output the corresponding optimal cargo throughput. These values are tabulated and presented in Table 5.

Cargo Throughput (LT/hr) For Each Optimal Preliminary Design Based on Delivery Distance							
Δ (LT)		1,000	2,000	3,000	4,000	5,000	6,000
V (kts)		40.7	36.6	33.8	31.1	29.4	28.0
R (nm)		1,337	2,168	2,801	3,501	4,002	4,497
P (LT)		162	320	478	631	788	943
Delivery Distance	500 nm	6.16	7.83	8.67	9.12	9.47	9.72
	1,000 nm	4.20	5.87	6.83	7.40	7.86	8.21
	1,500 nm	3.00	4.69	5.64	6.22	6.72	7.10
	2,000 nm	2.41	3.91	4.80	5.37	5.87	6.26
	2,500 nm	2.01	3.09	4.18	4.72	5.21	5.59
	3,000 nm	1.73	2.69	3.37	4.22	4.68	5.05
	3,500 nm	1.51	2.38	3.01	3.81	4.25	4.61
	4,000 nm	1.34	2.14	2.72	3.13	3.89	4.24
	4,500 nm	1.21	1.94	2.48	2.87	3.23	3.52
	5,000 nm	1.10	1.78	2.28	2.65	2.98	3.26
	5,500 nm	1.01	1.64	2.11	2.46	2.78	3.04
	6,000 nm	0.93	1.52	1.97	2.29	2.59	2.85
	6,500 nm	0.87	1.42	1.84	2.15	2.44	2.68
	7,000 nm	0.81	1.33	1.73	2.02	2.30	2.53

Table 5. Preliminary Design Optimal Solutions and Values.

In each column of Table 5 there is a double line separating two sets of cargo throughput values. This is the split between non-refueling and refueling operations. If the

round-trip delivery distance is less than that of the design ship's range then no refueling is needed, otherwise refueling is required. Of note, the approach in this study optimizes the solution based on the predominantly long delivery distance that would require refueling. Therefore, throughput values in the non-refueling section would be less than that of the design not taking refueling into consideration. Appendix M provides the MATLAB codes to produce the graphical results in Figure 21 in regard to the maximum cargo throughput for each optimal solution set (represented by the displacement value). The step down in every curve represents the shift from no refueling to refueling.

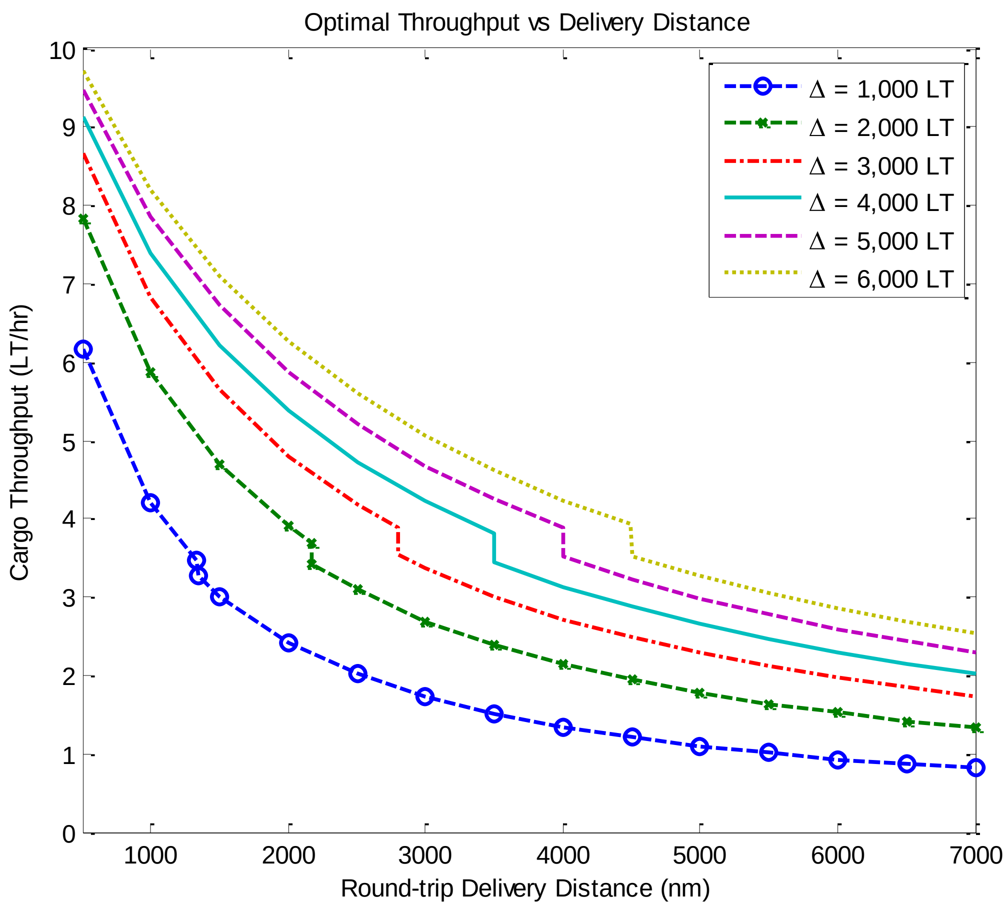

Figure 21. Cargo Throughput vs. Round-trip Delivery Distance for Each Optimal Design Solutions.

Appendix N provides the MATLAB codes for Figure 22, which gives the graphical presentation of the optimal solutions for speed, range, and payload (the three primary variables) based on a given design displacement. Though the solutions were calculated at a 1,000 LT increment of displacement, optimal solutions can easily be obtained via interpolations.

Recall the output of the mathematical model that displayed the relationship among three variables Figures 5–7 with the displacement fixed at 4,000 LT. The scenario can now be revisited with an optimal solution plotted to give its visual result comparing to the infinite combinations of design options. Figure 23 gives the result for a particular case, displacement of 4,000 LT, where the asterisk represents the optimal solution. This can be done for any displacement value by simply manipulating the data input into the program in Appendix E.

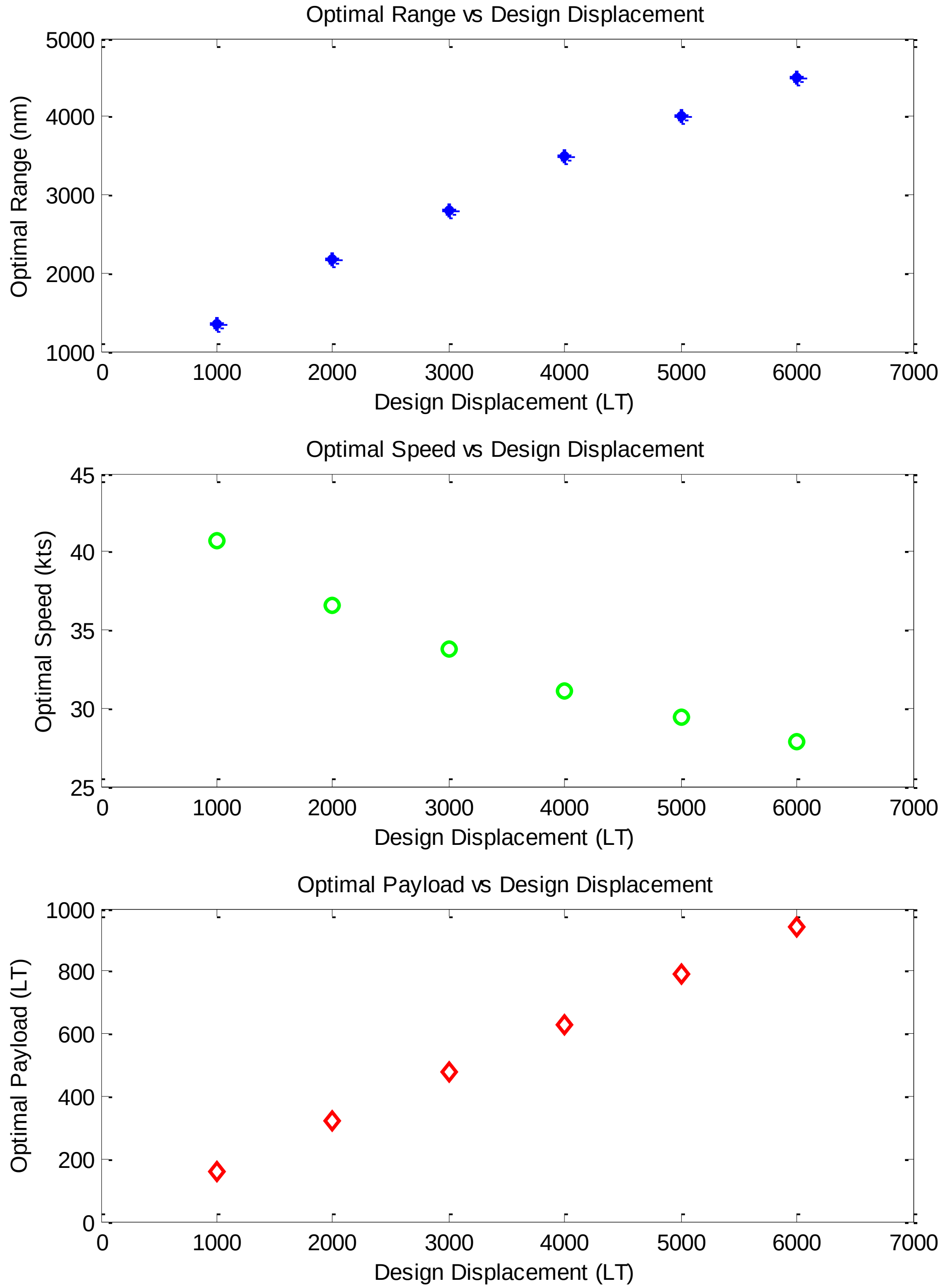

Figure 22. Design Optimal Solutions.

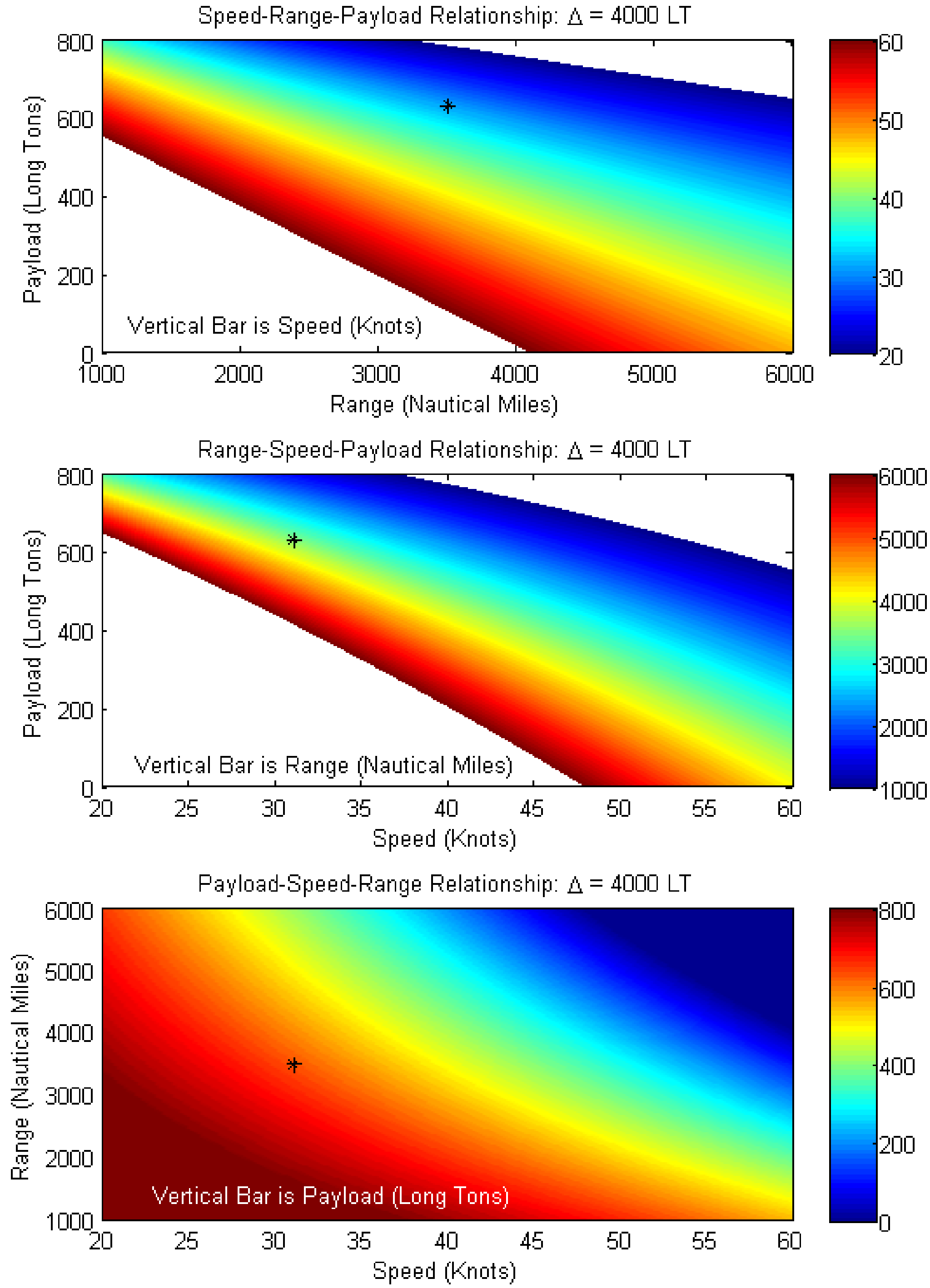

Figure 23. Mathematical Model Displayed with Optimal Solution for a 4,000 LT Design Displacement.

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VI. CONCLUSION

The results presented in this study are intended for a preliminary, not the final design. As such, this research is based upon a simple physics-based mathematical model. Therefore, the solutions in this study need to be taken in light of the following factors:

- Considerations in regard to cost, engine type/requirements, defense requirements, hull type, operational constraints, etc., are not included.
- Since the data used to input into the mathematical model came from the actual ships, the output represents the best of the combination of the ships used in this study, in a purely model sense.
- The output of the optimization problem relied on the accuracy of the data provided by A. S. Wright (delay time, loading/unloading times) [6], or at least their variations do not significantly impact the results.
- The output from the MATLAB optimization problem has a slight chance of not being the global optimal solution since the problem was not convex.
- The optimization problem assumed the ship is designed for cargo carriage requiring refueling (i.e., long distances).

In order to find the optimal solution that includes an optimal displacement value, a constraint on cost needs to be added to the optimization problem. Since there is a dollar figure associated with each ton of displacement and each horse power to produce the desired speed, a cost equation as a function of displacement and speed will help optimize the design displacement.

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APPENDIX A. HIGH SPEED VESSELS RAW DATA

VESSEL	Data				
	Displacement, Δ	Displacement Volume, ∇	Velocity, V	Velocity, V	Power, Ps
	(metric ton)	(m^3)	(knots)	(m/s)	(kW)
NEW ZEALAND: Protector-class offshore patrol [7]	1,900	1,854	22	11.32	10800
JAPAN: Takanami-class destroyer [8]	6,401	6,245	30	15.43	45000
SOUTH KOREA: Gwanggaeto the Great class destroyer [9]	3,900	3,805	30	15.43	59000
PAKISTAN: F-22P Zulfiquar-class frigate [10]	3,144	3,067	29	14.92	17100
BRAZIL: Niteroi-class frigate [11]	3,707	3,617	30	15.43	55000
TURKEY: Milgem-class Corvette [12]	2,300	2,244	30	15.43	31640
HST-1 [13]	1,646	1,606	35	18.01	32800
MV Fairweather [14]	760	741	32	16.46	11454
LCS-2 [15]	3,084	3,009	44	22.64	50120
JHSV [16]	2,362	2,304	43	22.12	36400
LCS-1 [17]	2,862	2,792	47	24.18	72000
Low-Wash Catamaran [18]	45	44	23.7	12.19	746
HSV-2 [19]	1,695	1,653	45	23.15	28800
INDONESIA: Klewang-class Trimaran [20]	219	214	35	18.01	7200
Bravest [21]	140	137	35	18.01	4000
RUSSIA: Vessel of A45 project [22]	68	66	38	19.55	2100
Gomel Polesye [23]	20	20	35	18.01	810
Gomel Byelorus [23]	15	14	34	17.49	708
Boeing Jetfoil 929-100 [24]	110	107	50	25.72	5534

MALAYSIA: 35.6 m Monohull [25]	25	24	40	20.58	1618
Patricia Olivia II [21]	202	197	57	29.32	11940

APPENDIX B. HSV CALCULATED DATA

VESSEL	Calculated	
	Fn	admiralty coefficient
	$(V/\sqrt{(g\nabla^{1/3})}$	$Ps = c\nabla^{2/3}V^3$
NEW ZEALAND: Protector-class offshore patrol	1.0310	49.369
JAPAN: Takanami-class destroyer	1.1482	36.098
SOUTH KOREA: Gwanggaeto the Great class destroyer	1.2471	65.854
PAKISTAN: F-22P Zulfiquar-class frigate	1.2496	24.394
BRAZIL: Niteroi-class frigate	1.2577	63.502
TURKEY: Milgem-class Corvette	1.3618	50.218
HST-1	1.6799	40.975
MV Fairweather	1.7470	31.340
LCS-2	1.9020	20.734
JHSV	1.9433	19.274
LCS-1	2.0572	25.688
Low-Wash Catamaran	2.0725	33.074
HSV-2	2.1494	16.601
INDONESIA: Klewang-class Trimaran	2.3512	34.512
Bravest	2.5332	25.837
RUSSIA: Vessel of A45 project	3.1021	17.153
Gomel Polesye	3.5036	19.145
Gomel Byelorus	3.5909	22.620
Boeing Jetfoil 929-100	3.7673	14.399
MALAYSIA: 35.6 m Monohull	3.8580	22.079
Patricia Olivia II	3.8810	13.983

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APPENDIX C. MATLAB CODES: ADMIRALTY COEFFICIENT

```
%% ADMIRALTY COEFFICIENT
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% This program determines the relationship between the admiralty  %
%   coefficient (c) and the Froude number (Fn) in the form of:     %
%              c = a + b*Fn^(-m)                                   %
%   where m is an integer from 1 to 4, with a and b are constants %
%    established by the best fit curve for each value of m.        %
%   The sum of Chi Square method is used to determine the value    %
%    of m that gives the least error to the generated curve.       %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clc
clear all
close all
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% The data for following High Speed Vessels (HSVs) is used %
%   (listed in the same order as the Fn and c vectors):    %
%    NEW ZEALAND: Protector-class Offshore Patrol           %
%    JAPAN: Takanami-class Destroyer                        %
%    SOUTH KOREA: Gwanggaeto the Great-class Destroyer      %
%    PAKISTAN: F-22P Zulfiquar-class Frigate                %
%    BRAZIL: Niteroi-class Frigate                          %
%    TURKEY: Milgem-class Corvette                          %
%    HST-1                                                  %
%    MV Fairweather                                         %
%    LCS-2                                                  %
%    JHSV                                                   %
%    LCS-1                                                  %
%    Low-Wash Catamaran                                     %
%    HSV-2                                                  %
%    INDONESIA: Klewang-class Trimaran                      %
%    Bravest                                                %
%    RUSSIA: Vessel of A45 Project                          %
%    Gomel Polesye                                          %
%    Gomel Byelorus                                         %
%    Boeing Jetfoil 920-100                                 %
%    MALAYSIA: 35.6 m Monohull                              %
%    Patricia Olivia II                                     %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Fn = [1.0310 1.1482 1.2471 1.2496 1.2577 1.3618 1.6799 1.7470 ...
   1.9020 1.9433 2.0572 2.0725 2.1494 2.3512 2.5332 3.1021 ...
   3.5036 3.5909 3.7673 3.8580 3.8810]';
c  = [49.369 36.098 65.854 24.394 63.502 50.218 40.975 31.340 ...
   20.734 19.274 25.688 33.074 16.601 34.512 25.837 17.153 ...
   19.145 22.620 14.399 22.079 13.983]';
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Testing for c as a function of (1/Fn) %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
x1     = Fn.^(-1);
```

```
xx      = linspace(0,4,100);
y1      = polyval(polyfit(x1,c,1),xx);
a1      = y1(1);
a1_str = num2str(a1);
b1      = (y1(100)-y1(1))/(xx(100)-xx(1));
b1_str = num2str(b1);
y11     = a1 + b1*xx.^(-1);
y12     = a1 + b1*x1;
%
Sum_Chi_1 = 0;
for i = 1:17
  Chi_1 = (c(i) - y12(i))^2/y12(i);
  Sum_Chi_1 = Sum_Chi_1 + abs(Chi_1);
end
SC1_str = num2str(Sum_Chi_1);
figure(1)
plot(Fn,c,'*',xx,y11,'LineWidth',1.5)
ylim([0 200])
xlabel('Volumetric Froude Number, Fn')
ylabel('Admiralty Coefficient, c')
title('Admiralty Coefficient (c) vs Froude Number (Fn)')
text(1.5,160,['c = ' a1_str ' + ' b1_str ' * Fn^{-1}'])
text(1.5,140,['Sum of Chi Square = ' SC1_str])
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Testing for c as a function of (1/Fn^2) %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
x2      = Fn.^(-2);
y2      = polyval(polyfit(x2,c,1),xx);
a2      = y2(1);
a2_str = num2str(a2);
b2      = (y2(100)-y2(1))/(xx(100)-xx(1));
b2_str = num2str(b2);
y22     = a2 + b2*xx.^(-2);
y21     = a2 + b2*x2;
%
Sum_Chi_2 = 0;
for i = 1:17
  Chi_2 = (c(i) - y21(i))^2/y21(i);
  Sum_Chi_2 = Sum_Chi_2 + Chi_2;
end
SC2_str = num2str(Sum_Chi_2);
figure(2)
plot(Fn,c,'*',xx,y22,'LineWidth',1.5)
ylim([0 200])
xlabel('Volumetric Froude Number, Fn')
ylabel('Admiralty Coefficient, c')
title('Admiralty Coefficient (c) vs Froude Number (Fn)')
text(1.5,160,['c = ' a2_str ' + ' b2_str ' * Fn^{-2}'])
text(1.5,140,['Sum of Chi Square = ' SC2_str])
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Testing for c as a function of (1/Fn^3) %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
x3      = Fn.^(-3);
```

```
y3     = polyval(polyfit(x3,c,1),xx);
a3     = y3(1);
a3_str = num2str(a3);
b3     = (y3(100)-y3(1))/(xx(100)-xx(1));
b3_str = num2str(b3);
y33    = a3 + b3*xx.^(-3);
y31    = a3 + b3*x3;
%
Sum_Chi_3 = 0;
for i = 1:17
  Chi_3 = (c(i) - y31(i))^2/y31(i);
  Sum_Chi_3 = Sum_Chi_3 + Chi_3;
end
SC3_str = num2str(Sum_Chi_3);
figure(3)
plot(Fn,c,'*',xx,y33,'LineWidth',1.5)
ylim([0 200])
xlabel('Volumetric Froude Number, Fn')
ylabel('Admiralty Coefficient, c')
title('Admiralty Coefficient (c) vs Froude Number (Fn)')
text(1.5,160,['c = ' a3_str ' + ' b3_str ' * Fn^{-3}'])
text(1.5,140,['Sum of Chi Square = ' SC3_str])
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Testing for c as a function of (1/Fn^4) %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
x4     = Fn.^(-4);
y4     = polyval(polyfit(x4,c,1),xx);
a4     = y4(1);
a4_str = num2str(a4);
b4     = (y4(100)-y4(1))/(xx(100)-xx(1));
b4_str = num2str(b4);
y44    = a4 + b4*xx.^(-4);
y41    = a4 + b4*x4;
%
Sum_Chi_4 = 0;
for i = 1:17
  Chi_4 = (c(i) - y41(i))^2/y41(i);
  Sum_Chi_4 = Sum_Chi_4 + Chi_4;
end
SC4_str = num2str(Sum_Chi_4);
figure(4)
plot(Fn,c,'*',xx,y44,'LineWidth',1.5)
ylim([0 200])
xlabel('Volumetric Froude Number, Fn')
ylabel('Admiralty Coefficient, c')
title('Admiralty Coefficient (c) vs Froude Number (Fn)')
text(1.5,160,['c = ' a4_str ' + ' b4_str ' * Fn^{-4}'])
text(1.5,140,['Sum of Chi Square = ' SC4_str])
```

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APPENDIX D. MATLAB CODES: DISPLACEMENT CALCULATIONS

```
%% MATHEMATICAL MODEL: DISPLACEMENT CALCULATIONS
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% This program calculates the required displacement for a High %
%   Speed Vessel (HSV) based on the input values for range,    %
%   speed, and payload.                                        %
%     R (Range) in nautical miles                              %
%     V (Speed) in knots                                       %
%     P (Payload) in long tons                                 %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clc
clear all
close all
%
R = 3512;
V = 45;
P = 400;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% a & b from the Admiralty Coefficient vs Volumetric Froude %
%   Number relationship: c = a + b * Fn^-1.                 %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
a = 3.3892;
b = 51.8489;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% alpha is the Cargo Carriage Multiplier:               %
%   W(cargo) = alpha * Payload, unit is lbs/lb or LT/LT %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
alpha = 4;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% beta is the Weight of Propulsion: W(prop) = beta * Ps, %
%   unit is in lbs/hp                                    %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
beta = 10;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% SFC is specific fuel consumption, unit is lb/hp-hr %
% g is acceleration due to gravity in m/s^2          %
% rho is density of seawater in kg/m^3               %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
SFC = 0.4;
G   = 9.81;
rho = 1025;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Converting into standard metric units (kg, m, s)         %
%  lbs/hp to kg/W, lbs/hp-hr to kg/W-s, metric tons to kg, %
%  knots to m/s, nautical miles to meter                   %
%   1 hp = 745.7 W, 1 lb = 0.4536 kg, 1 hr = 3600 sec,     %
%   1 LT = 1016.05 kg, 1 nmi = 1852 m, 1 kts = 0.5144 m/s  %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
beta1 = beta*0.4536/745.7;
SFC1  = SFC*0.4536/(745.7*3600);
```

```
R1    = R*1852;
V1    = V*0.5144;
P1    = P*1016.05;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Do is the first guess for the displacement for iteration %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Do = 2000*1016.05;
%%%%%%%%%%%%%%%%%%%%%%%%
% Performing iteration %
%%%%%%%%%%%%%%%%%%%%%%%%
N = 100;
j = 1;
while j <= N
  D = alpha*P1 + (SFC1*R1/V1 + beta1)*(a + b*g^(1/2)* ...
    (Do/rho)^(1/6)/V1)*(Do/rho)^(2/3)*V1^3;
  if abs(D-Do) < 1
  end
  j = j+1;
  Do = D;
end
D1 = D/1016.05;
fprintf(‘\n  Speed   Range   Payload  Displacement’)
fprintf(‘\n  (kts)   (nmi)    (LT)     (LT)\n’)
fprintf(‘\n  %4.0f   %4.0f    %4.0f     %4.0f\n\n’,...
    V,R,P,D1)
```

APPENDIX E. MATLAB CODES: THE MATHEMATICAL MODEL

```
%% MATHEMATICAL MODEL: SPEED-RANGE-PAYLOAD RELATIONSHIP
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% This program produces the inter-relationship amongst Payload, %
%   Range, and Speed. Three graphs are to be generated,         %
%   alternating the position of the variables on the graph.     %
%   The required input is displacement.                         %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clc
clear all
close all
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Assumes a displacement of 4,000 LT %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
D = 4000;
D_string = num2str(D);
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Constants:                                                    %
%   a & b - from the Admiralty Coefficient vs Volumetric Froude %
%    Number relationship: c = a + b * Fn^-1, unit is kg/m^3.    %
%   alpha - the Cargo Carriage Multiplier:                      %
%    W(cargo) = alpha * Payload, unit is lbs/lb or LT/LT        %
%   beta - the Weight of Propulsion: W(prop) = beta * Ps,       %
%    unit is lbs/hp                                             %
%   SFC - specific fuel consumption, unit is lb/hp-hr           %
%   g   - acceleration due to gravity in m/s^2                  %
%   rho - density of seawater in kg/m^3                         %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
a     = 3.3892;
b     = 51.8489;
alpha = 4;
beta  = 10;
SFC   = 0.4;
g     = 9.81;
rho   = 1025;
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Converting lbs/hp to kg/W, lbs/hp-hr to kg/W-s, and metric tons %
%   to kg:                                                        %
%   1 hp = 745.7 W, 1 lb = 0.4536 kg, 1 hr = 3600 sec,            %
%   1 LT = 1016.05 kg.                                            %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
beta1 = beta*0.4536/745.7;
SFC1  = SFC*0.4536/745.7/3600;
D1    = D*1016.05;
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Equate & plot the relationship Using Equation 16. %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
[V,R] = meshgrid(linspace(20,60,200),linspace(1000,6000,200));
P = (D1 - (SFC1.*(R*1852)./(V*0.5144) + beta1).*(a + b*g^(1/2)* ...
  (D1/rho)^(1/6)./(V*0.5144)).*(D1/rho)^(2/3).*(V*0.5144).^3)/ ...
  (alpha*1016.05);
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Display Payload vs Range plot with Speed on a side vertical bar %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
figure(1)
pcolor(R,P,V),caxis([20 60])
xlabel('Range (Nautical Miles)')
ylabel('Payload (Long Tons)')
title(['Speed-Range-Payload Relationship: \Delta = ' D_string,' LT'])
xlim([1000 6000])
ylim([0 800])
text(1200,75,'Vertical Bar is Speed (Knots)')
shading interp
colorbar
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Display Payload vs Speed plot with Range on a side vertical bar %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
figure(2)
pcolor(V,P,R),caxis([1000 6000])
xlabel('Speed (Knots)')
ylabel('Payload (Long Tons)')
title(['Range-Speed-Payload Relationship: \Delta = ' D_string,' LT'])
xlim([20 60])
ylim([0 800])
text(22,60,'Vertical Bar is Range (Nautical Miles)')
shading interp
colorbar
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Display Range vs Speed plot with Payload on a side vertical bar %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
figure(3)
pcolor(V,R,P),caxis([0 800])
xlabel('Speed (Knots)')
ylabel('Range (Nautical Miles)')
title(['Payload-Speed-Range Relationship: \Delta = ' D_string,' LT'])
xlim([20 60])
ylim([1000 6000])
text(23,1400,'Vertical Bar is Payload (Long Tons)','Color','white')
shading interp
colorbar
```

APPENDIX F. MATLAB CODES: CONSTANTS CALCULATIONS—PART 1

```
%% MATHEMATICAL MODEL: CONSTANTS AS FUNCTION OF DISPLACEMENT
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% This program determines the constant values of a1, a2, b1, and %
%   b2 as a function of displacement for the Payload vs. Range   %
%   relationship.                                                %
%     P = A + B * R                                              %
%       A = a1 + a2*V^2 + a3*V^3                                 %
%       B = b1*V + b2*V^2                                        %
%   where P- payload (LT), R- range (nautical miles), and        %
%      V- speed (knots)                                          %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clc
clear all
close all
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Constants:                                                    %
%   a & b - from the Admiralty Coefficient vs Volumetric Froude %
%    Number relationship: c = a + b * Fn^-1, unit is kg/m^3.    %
%   alpha - the Cargo Carriage Multiplier:                      %
%    W(cargo) = alpha * Payload, unit is lbs/lb or LT/LT        %
%   beta - the Weight of Propulsion: W(prop) = beta * Ps,       %
%    unit is lbs/hp                                             %
%   SFC  - specific fuel consumption, unit is lb/hp-hr          %
%   g   - acceleration due to gravity in m/s^2                  %
%   rho  - density of seawater in kg/m^3                        %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
a     = 3.3892;
b     = 51.8489;
alpha = 4;
beta  = 10;
SFC   = 0.4;
g     = 9.81;
rho   = 1025;
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Converting lbs/hp to kg/W, lbs/hp-hr to kg/W-s, and metric tons %
%   to kg:                                                        %
%   1 hp = 745.7 W, 1 lb = 0.4536 kg, 1 hr = 3600 sec,            %
%   1 LT = 1016.05 kg, 1 kts = 0.5144 m/s                         %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
beta1 = beta*0.4536/745.7;
SFC1  = SFC*0.4536/745.7/3600;
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Establish a1, a2, b1, b2 as functions of displacement %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
D  = zeros(1,9);
D1 = zeros(1,9);
```

```
a1 = zeros(1,9);
a2 = zeros(1,9);
a3 = zeros(1,9);
b1 = zeros(1,9);
b2 = zeros(1,9);
for i = 1:11
  D(i) = 500 + 500*i;
  D1(i) = D(i)*1016.05;
  a1(i) = D1(i)/(alpha*1016.05);
  a2(i) = -beta1*b*g^(1/2)*(D1(i)/rho)^(5/6)*(0.5144)^2/(alpha* ...
      1016.05);
  a3(i) = -beta1*a*(D1(i)/rho)^(2/3)*(0.5144)^3/(alpha*1016.05);
  b1(i) = -SFC1*b*g^(1/2)*(D1(i)/rho)^(5/6)*1852*0.5144/(alpha* ...
      1016.05);
  b2(i) = -SFC1*a*(D1(i)/rho)^(2/3)*1852*(0.5144)^2/(alpha*1016.05);
end
%
%%%%%%%%%%%%%%%%%%%%%
% Print the results %
%%%%%%%%%%%%%%%%%%%%%
fprintf('\n A = a1 + a2*V^2 + a3*V^3  AND  B = b1*V + b2*V^2')
fprintf('\n\n P = A + B * R ;  where V: Speed (kts), P: Payload(LT), R:
Range(nmi)\n')
fprintf('\n Displacement   a1      a2       a3       b1       b2')
fprintf('\n   (LT)')
fprintf('\n   %4.0f      %4.1f    %1.4f    %1.6f    %1.5f    %1.8f\n', ...
   [D; a1; a2; a3; b1; b2])
%
%%%%%%%%%%%%%%%%%%%%%%%%%%
% Plot the relationships %
%%%%%%%%%%%%%%%%%%%%%%%%%%
figure('Position', [0 0 800 1200]);
subplot(311)
plot(D,a1)
grid
xlabel('Displacement, \Delta (LT)')
ylabel('a_1')
title('a_1 as a function of \Delta')
subplot(312)
plot(D,a2)
grid
xlabel('Displacement, \Delta (LT)')
ylabel('a_2')
title('a_2 as a function of \Delta')
subplot(313)
plot(D,a3)
grid
xlabel('Displacement, \Delta (LT)')
ylabel('a_3')
title('a_3 as a function of \Delta')
%
figure('Position', [0 0 800 800]);
subplot(211)
plot(D,b1)
grid
```

```
xlabel('Displacement, \Delta (LT)')
ylabel('b_1')
title('b_1 as a function of \Delta')
subplot(212)
plot(D,b2)
grid
xlabel('Displacement, \Delta (LT)')
ylabel('b_2')
title('b_2 as a function of \Delta')
```

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APPENDIX G. MATLAB CODES: CONSTANTS CALCULATIONS—PART 2

```
%% MATHEMATICAL MODEL: CONSTANTS AS FUCNTION OF SPEED
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% This program determines the values of constants A and B as          %
%   functions of speed based on a specific input for displacement. %
%    P = A(V) + B(V)*R                                                %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clc
clear all
close all
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Assumed a displacement of 4,000 LT %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
D = 4000;
D_string = num2str(D);
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Constants:                                                       %
%   a & b - from the Admiralty Coefficient vs Volumetric Froude %
%    Number relationship: c = a + b * Fn^-1, unit is kg/m^3.    %
%   alpha - the Cargo Carriage Multiplier:                         %
%    W(cargo) = alpha * Payload, unit is lbs/lb or LT/LT          %
%   beta - the Weight of Propulsion: W(prop) = beta * Ps,         %
%    unit is lbs/hp                                                %
%   SFC  - specific fuel consumption, unit is lb/hp-hr            %
%   g    - acceleration due to gravity in m/s^2                   %
%   rho  - density of seawater in kg/m^3                          %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
a     = 3.3892;
b     = 51.8489;
alpha = 4;
beta  = 10;
SFC   = 0.4;
g     = 9.81;
rho   = 1025;
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Converting lbs/hp to kg/W, lbs/hp-hr to kg/W-s, and metric tons %
%   to kg:                                                           %
%   1 hp = 745.7 W, 1 lb = 0.4536 kg, 1 hr = 3600 sec,              %
%   1 LT = 1016.05 kg, 1 kts = 0.5144 m/s                           %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
beta1 = beta*0.4536/745.7;
SFC1  = SFC*0.4536/745.7/3600;
D1    = D*1016.05;
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%  Establish P = A + B * R Relationship  %
%     A = A(V) and B = B(V)              %
```

```matlab
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
R  = linspace(0,10000,50);
a1 = D1/(alpha*1016.05);
a2 = -beta1*b*g^(1/2)*(D1/rho)^(5/6)*(0.5144)^2/(alpha*1016.05);
a3 = -beta1*a*(D1/rho)^(2/3)*(0.5144)^3/(alpha*1016.05);
b1 = -SFC1*b*g^(1/2)*(D1/rho)^(5/6)*1852*0.5144/(alpha*1016.05);
b2 = -SFC1*a*(D1/rho)^(2/3)*1852*(0.5144)^2/(alpha*1016.05);
A  = zeros(1,9);
B  = zeros(1,9);
V  = zeros(1,9);
for i = 1:9
    V(i) = 15 + 5*i;
    A(i) = a1 + a2*V(i)^2 + a3*V(i)^3;
    B(i) = b1*V(i) + b2*V(i)^2;
end
P1 = A(1) + B(1)*R;
P2 = A(2) + B(2)*R;
P3 = A(3) + B(3)*R;
P4 = A(4) + B(4)*R;
P5 = A(5) + B(5)*R;
P6 = A(6) + B(6)*R;
P7 = A(7) + B(7)*R;
P8 = A(8) + B(8)*R;
P9 = A(9) + B(9)*R;
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%  Plot the relationships   %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
figure('Position', [0 0 700 500]);
plot(R,P1,R,P2,'--',R,P3,':',R,P4,':s',R,P5,':+',R,P6,':>',R,P7, ...
    ':o',R,P8,':d',R,P9,':p','LineWidth',1.5)
grid
xlabel('Range (Nautical Miles)')
ylabel('Payload (Long Tons)')
title(['Payload vs Range for Various Speeds (\Delta = ' D_string, ...
       ' LT)'])
legend('V = 20 kts','V = 25 kts','V = 30 kts','V = 35 kts', ...
       'V = 40 kts','V = 45 kts','V = 50 kts','V = 55 kts', ...
       'V = 60 kts',3)
xlim([1000 6000])
ylim([0 800])
%
figure('Position', [0 0 700 1000]);
subplot(211)
plot(V,A,'LineWidth',1.5)
grid
xlabel('Speed (knots)')
ylabel('A (y-intercept of P vs R relationship)')
title(['A(V) vs Speed (\Delta = ' D_string,' LT)'])
subplot(212)
plot(V,B,'LineWidth',1.5)
grid
xlabel('Speed (knots)')
ylabel('B (slope of P vs R relationship)')
title(['B(V) vs Speed (\Delta = ' D_string,' LT)'])
```

```
%
%%%%%%%%%%%%%%%%%%%%%%%
%  Print the results  %
%%%%%%%%%%%%%%%%%%%%%%%
fprintf('\n  P = A + B * R  ;  P- Payload(LT), R- Range(nmi)')
fprintf('\n                    Displacement = %4.0f Long Tons\n',D)
fprintf('\n   Speed       A           B')
fprintf('\n   (kts)')
fprintf('\n    %2.0f       %4.1f      %5.4f\n',[V; A; B])
```

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APPENDIX H. MATLAB CODES: THROUGHPUT OPTIMIZATION

```
%% THROUGHPUT OPTIMIZATION
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% This program produces the optimal solution (range, speed, and %
%   payload) and the optimal value (cargo throughput). It is    %
%   accomplished in the following steps.                        %
%   1. Using displacement of 4,000 LT and no refueling, it      %
%    produces the relationship between range and optimal speed. %
%   2. Using displacement of 4,000 LT with various refueling    %
%    rates, it produces a set of relationship between range and %
%    and optimal speed for particular refueling rates.          %
%   3. Once the optimal payload and range values are determined %
%    for a particular refueling rate, it can be used to         %
%    determine the optimal speed and cargo throughput.          %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clc
clear all
close all
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Parameters a, b, alpha, beta, SFC, g, rho, S, k               %
%   Only 2 of the above parameters can be changed: S & k.       %
%   For no refueling, choose S (roundtrip delivery distance) to %
%    be less than the lower bound of the range interval.        %
%    Ex: 400 nm.                                                %
%   With refueling, choose S to be greater than the upper bound %
%    of the range interval. Ex: 7,000 nm.                       %
%   Vary k based on the refueling rate.                         %
%   Once the optimal speed and range are determined,            %
%    incrementally vary S to get the cargo throughput as a      %
%    function of displacement and delivery distance.            %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
a     = 3.3892;
b     = 51.8489;
alpha = 4;
beta  = 10;
SFC   = 0.4;
g     = 9.81;
rho   = 1025;
S     = 7000;
%k    = 0;              % No refueling
%
%k    = 1.007E-4;       % Refueling at 1,000 gal/min for 1,000 LT
%k    = 5.034E-5;       % Refueling at 2,000 gal/min for 1,000 LT
%k    = 2.517E-5;       % Refueling at 4,000 gal/min for 1,000 LT
%k    = 1.259E-5;       % Refueling at 8,000 gal/min for 1,000 LT
%
%k    = 1.775E-4;       % Refueling at 1,000 gal/min for 2,000 LT
%k    = 8.873E-5;       % Refueling at 2,000 gal/min for 2,000 LT
%k    = 4.436E-5;       % Refueling at 4,000 gal/min for 2,000 LT
%k    = 2.218E-5;       % Refueling at 8,000 gal/min for 2,000 LT
```

```
%
%k    = 2.473E-4;      % Refueling at 1,000 gal/min for 3,000 LT
%k    = 1.237E-4;      % Refueling at 2,000 gal/min for 3,000 LT
%k    = 6.183E-5;      % Refueling at 4,000 gal/min for 3,000 LT
%k    = 3.092E-5;      % Refueling at 8,000 gal/min for 3,000 LT
%
%k    = 3.131E-4;      % Refueling at 1,000 gal/min for 4,000 LT
k     = 1.566E-4;      % Refueling at 2,000 gal/min for 4,000 LT
%k    = 7.828E-5;      % Refueling at 4,000 gal/min for 4,000 LT
%k    = 3.914E-5;      % Refueling at 8,000 gal/min for 4,000 LT
%
%k    = 3.760E-4;      % Refueling at 1,000 gal/min for 5,000 LT
%k    = 1.880E-4;      % Refueling at 2,000 gal/min for 5,000 LT
%k    = 9.400E-5;      % Refueling at 4,000 gal/min for 5,000 LT
%k    = 4.700E-5;      % Refueling at 8,000 gal/min for 5,000 LT
%
%k    = 4.367E-4;      % Refueling at 1,000 gal/min for 6,000 LT
%k    = 2.183E-4;      % Refueling at 2,000 gal/min for 6,000 LT
%k    = 1.092E-4;      % Refueling at 4,000 gal/min for 6,000 LT
%k    = 5.458E-5;      % Refueling at 8,000 gal/min for 6,000 LT
parameters = [a; b; alpha; beta; SFC; g; rho; S; k];
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Boundaries (payload, range, speed, displacement)                %
%   Since larger displacement leads to higher cargo throughput,   %
%    the program will always maximize the displacement value      %
%    to its upper bound. Therefore, choose a particular           %
%    displacement for example 4,000 LT, to evaluate the other     %
%    variables.                                                   %
%   Incrementally increase the range from its 500 nm lower bound  %
%    value to see the change in the optimal speed.                %
%   Once the optimal speed and range are determined, payload and  %
%    throughput can be found by varying the design                %
%    displacement.                                                %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
lb = [100;1000;15;1000];
ub = [1300;6000;60;6000];
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Design guess (payload, range, speed, displacement)             %
%   Since this optimization problem is not convex, choose the    %
%    initial guess that makes the most sense in optimizing the   %
%    throughput.                                                 %
%   Maximizing throughput means maximizing the payload,          %
%    minimizing the range, maximizing the speed, and maximizing  %
%    the displacement.                                           %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
x = [ub(1) lb(2) ub(3) ub(4)];
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Calling the objective function %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
obj_fun = @(x) TP_Obj_Fun(parameters,x);
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
% Calling the constraint function %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
nonlincon = @(x) TP_Cons_Fun(parameters,x);
%
%%%%%%%%%%%%%%%%%%%%%%%%%%
% Setting solver options %
%%%%%%%%%%%%%%%%%%%%%%%%%%
options = optimset('Algorithm','interior-point','Display','off', ...
  'MaxIter',5000,'MaxFunEvals',30000,'TolFun',1e-7,'TolCon',1e-7, ...
  'LargeScale','off','GradObj','on','GradConstr','on', ...
  'DerivativeCheck','off');
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Find the optimal solution using fmincon %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
[x_opt,obj_opt] = fmincon(obj_fun,x,[],[],[],[],lb,ub,nonlincon, ...
         options);
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Find the optimal value (cargo throughput) %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
TP = -obj_opt;
%
%%%%%%%%%%%%%%%%%%%%%
% Print the results %
%%%%%%%%%%%%%%%%%%%%%
fprintf('\n\nThe optimal ship design variables are:\n\n')
fprintf('  Payload (LT)  Range (nm)  Speed (kts)  Displacement (LT)\n')
fprintf('    %3.0f      %4.0f      %2.2f      %4.0f',x_opt)
fprintf('\n\nThe optimal throughput based on the above design is:\n')
fprintf('\n    %2.2f LT/hr\n',TP)


%% THROUGHPUT OPTIMIZATION: OBJECTIVE FUNCTION
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% This program generates the objective function based on the  %
%   throughput equation. Its gradients are calculated for the %
%   fmincon function to be used in the main program           %
%   TP_Throughput_Optimization.m                              %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
function [f_val,f_grad] = TP_Obj_Fun(parameters,x)

S = parameters(8);
k = parameters(9);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Objective function and its gradient %
%   x(1) is payload                   %
%   x(2) is range                     %
%   x(3) is speed                     %
%   x(4) is displacement              %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
f_val = -x(1) / (S/x(3) + 0.5 + x(1)/12 + k*S*x(3));
```

```
f_grad = [ x(1)/(12*(x(1)/12 + S/x(3) + S*k*x(3) + 1/2)^2) - ...
      1/(x(1)/12 + S/x(3) + S*k*x(3) + 1/2);
      0;
      (x(1)*(S*k - S/x(3)^2))/(x(1)/12 + S/x(3) + S*k*x(3) + ...
      1/2)^2;
      0];


%% THROUGHPUT OPTIMIZATION: CONSTRAINT FUNCTION
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% This program generates the constraint function based on     %
%   Equation 16. Its gradients are calculated for the fmincon %
%    function in various programs.                            %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
function [h,heq,grad_h,grad_heq] = TP_Cons_Fun(parameters,x)

a      = parameters(1);
b      = parameters(2);
alpha = parameters(3);
beta  = parameters(4);
SFC   = parameters(5);
g      = parameters(6);
rho   = parameters(7);
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Converting lbs/hp to kg/W, lbs/hp-hr to kg/W-s, and metric tons %
% to kg: 1 hp = 745.7 W, 1 lb = 0.4536 kg, 1 hr = 3600 sec        %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
beta1 = beta*0.4536/745.7;
SFC1  = SFC*0.4536/745.7/3600;
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% No inequality constraints %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%
h       = [];
grad_h = [];
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Equality Constraint Function and its gradient %
%   x(1) is payload                              %
%   x(2) is range                                %
%   x(3) is speed                                %
%   x(4) is displacement                         %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
heq = 1016.05*alpha*x(1) - 1016.05*x(4) + (SFC1*(x(2)*1852)/(x(3)* ...
   0.5144) + beta1)*(a + b*g^(1/2)*((1016.05*x(4))/rho)^(1/6)/ ...
   (x(3)*0.5144))*((1016.05*x(4))/rho)^(2/3)*(x(3)*0.5144)^3;
%
grad_heq = [ (20321*alpha)/20; ...
  (191426887*SFC1*x(3)^2*(a + (1250*b*g^(1/2)*((20321*x(4)) ...
  /(20*rho))^(1/6))/(643*x(3)))*((20321*x(4))/(20*rho))^(2/3))/ ...
```

```
390625; ...
(797543121*x(3)^2*(beta1 + (2315000*x(2)*SFC1)/(643*x(3)))*(a ...
+ (1250*b*g^(1/2)*((20321*x(4))/(20*rho))^(1/6))/(643*x(3))) ...
*((20321*x(4))/(20*rho))^(2/3))/1953125000 - (191426887*x(2)* ...
SFC1*x(3)*(a + (1250*b*g^(1/2)*((20321*x(4))/(20*rho))^(1/6))/ ...
(643*x(3)))*((20321*x(4))/(20*rho))^(2/3))/390625 - (413449* ...
x(3)*b*g^(1/2)*(beta1 + (2315000*x(2)*SFC1)/(643*x(3)))* ...
((20321*x(4))/(20*rho))^(5/6))/1562500; ...
(5402291253947*x(3)^3*(beta1 + (2315000*x(2)*SFC1)/(643*x(3))) ...
*(a + (1250*b*g^(1/2)*((20321*x(4))/(20*rho))^(1/6))/(643*x(3)))) ...
/(58593750000*rho*((20321*x(4))/(20*rho))^(1/3)) + (8401697129 ...
*x(3)^2*b*g^(1/2)*(beta1 + (2315000*x(2)*SFC1)/(643*x(3)))) ...
/(187500000*rho*((20321*x(4))/(20*rho))^(1/6)) - 20321/20 ];
```

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APPENDIX I. MATLAB CODES: OPTIMAL SPEED AND PAYLOAD VERSUS RANGE

```
%% OPTIMAL SPEED AND PAYLOAD VS RANGE
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% This program plots the results of the MATLAB program             %
%   TP_Throughput_Optimization.m, showing:                         %
%   1. The optimal speed based on range for several different      %
%     refueling values at a design displacement of 4,000 LT.       %
%   2. The optimal speed based on range for several different      %
%     design displacements at a refueling rate of 2,000 gal/min.   %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clc
clear all
close all
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Data from optimization program                                  %
%   R = ship's range in nautical miles                            %
%   Vi_D = optimal speed with refueling at i*1000 gallons per     %
%      minute with constant displacement of 4,000 LT              %
%   Vi_k = optimal speed with displacement at i*1000 LT with      %
%      constant refueling rate at 2,000 gallons per minute.       %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
R  = [500 1000 1500 2000 2500 3000 3500 4000 4500 5000 5500 6000];
%
V0_D = [57.68 52.61 48.12 44.15 40.65 37.57 34.84 32.42 30.27 28.36 ...
    26.65 25.11];
V1_D = [39.28 37.09 35.04 33.13 31.35 29.70 28.17 26.75 25.43 24.21 ...
    23.08 22.03];
V2_D = [45.88 42.86 40.07 37.51 35.16 33.02 31.05 29.26 27.62 26.13 ...
    24.76 23.50];
V4_D = [51.00 47.21 43.77 40.64 37.82 35.27 32.97 30.89 29.02 27.32 ...
    25.79 24.40];
V8_D = [54.45 50.08 46.15 42.62 39.46 36.63 34.11 31.85 29.82 28.01 ...
    26.37 24.90];
%
P0_D = [668.2 643.6 623.5 607.1 593.7 582.5 573.3 565.7 559.2 553.7 ...
    548.9 545];
P1_D = [835.0 798.5 766.6 738.5 714.0 692.5 673.6 657.1 642.6 629.8 ...
    618.5 608.5];
P2_D = [782.4 746.2 715.4 689.2 666.8 647.7 631.5 617.5 605.5 595.1 ...
    586.1 578.5];
P4_D = [736.0 702.7 674.7 651.6 632.2 616.0 602.4 591.0 581.3 573.1 ...
    566.0 559.8];
P8_D = [702.1 671.9 647.2 626.8 610.1 596.3 584.7 575.2 567.2 560.2 ...
    554.5 549.4];
%
%%%%%%%%%%%%%%%%%%%%
% Plot the results %
%%%%%%%%%%%%%%%%%%%%
figure('Position', [0 0 700 600]);
```

```
plot(R,V0_D,R,V1_D,':o',R,V2_D,':*',R,V4_D,'--',R,V8_D,':', ...
    'LineWidth',1.5)
xlim([0,6500])
xlabel('Range (nm)')
ylabel('Speed (kts)')
title('Optimal Speed vs Range for Various Refueling Rates')
text(1000,25,'\Delta = 4,000 LT')
legend('delivery distance < range (no refueling)', ...
    '1000 gal/min refueling rate','2000 gal/min refueling rate', ...
    '4000 gal/min refueling rate','8000 gal/min refueling rate')
%
figure('Position', [0 0 700 600]);
plot(R,P0_D,R,P1_D,':*',R,P2_D,':o',R,P4_D,'--',R,P8_D,':', ...
    'LineWidth',1.5)
xlim([0,6500])
xlabel('Range (nm)')
ylabel('Payload (LT)')
title('Optimal Payload vs Range for Refueling Rates')
text(1000,550,'\Delta = 4,000 LT')
legend('delivery distance < range (no refueling)', ...
    '1000 gal/min refueling rate','2000 gal/min refueling rate', ...
    '4000 gal/min refueling rate','8000 gal/min refueling rate')
```

APPENDIX J. MATLAB CODES: α SENSITIVITY ANALYSIS

```
%% SENSITIVITY ANALYSIS: CARGO CARRIAGE MULTIPLIER
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% This program finds optimal primary variable(s) that is least %
%   sensitive to the change in the cargo carriage multiplier.  %
% Displacement = 4,000 LT                                      %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clc
clear all
close all
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Data from Optimization Program based on different values of alpha %
%   R = range (nm)                                                  %
%   V = optimal speed for alpha equals i                            %
%   P_i = optimal payload for alpha equals i                        %
%   TP_i = optimal throughput for alpha equals i                    %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
R  = [500 1000 1500 2000 2500 3000 3500 4000 4500 5000 5500 6000];
%
V  = [45.88 42.86 40.07 37.51 35.16 33.02 31.05 29.26 27.62 26.13 ...
    24.76 23.50];
%
P_2 = [1564 1492 1430 1378 1333 1295 1262 1234 1210 1189 1171 1156];
P_4 = [ 782 746 715 689 667 648 632 618 606 595 586 579];
%
TP_2 = [4.69 4.45 4.23 4.02 3.82 3.63 3.46 3.29 3.14 3.00 2.87 2.75];
TP_4 = [2.91 2.73 2.57 2.41 2.27 2.14 2.02 1.91 1.81 1.72 1.63 1.55];
%
%%%%%%%%%%%%%%%%%%%%
% Plot the results %
%%%%%%%%%%%%%%%%%%%%
plot3(R,V,P_2,R,V,P_4,'--','LineWidth',1.5)
grid
xlabel('Range (nm)')
ylabel('Speed (kts)')
zlabel('Payload (LT)')
title('Sensitivity Analysis - Cargo Carriage Multiplier (\alpha)')
zlim([0 1600])
legend('\alpha = 2','\alpha = 4')
```

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APPENDIX K. MATLAB CODES: β SENSITIVITY ANALYSIS

```
%% SENSITIVITY ANALYSIS: WEIGHT OF POWER FACTOR
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% This program finds optimal primary variable(s) that is least %
%   sensitive to the change in the weight of power factor.     %
% Displacement = 4,000 LT                                      %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clc
clear all
close all
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Data from Optimization Program based on different value of beta %
%   R = range (nm)                                                 %
%   V_i = optimal speed for beta equals i                          %
%   P_i = optimal payload for beta equals i                        %
%   TP_i = optimal throughput for beta equals i                    %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
R    = [500 1000 1500 2000 2500 3000 3500 4000 4500 5000 5500 6000];
%
V_5  = [53.45 49.19 45.36 41.93 38.87 36.12 33.67 31.47 29.50 27.73 ...
    26.13 24.68];
V_10 = [45.88 42.86 40.07 37.51 35.16 33.02 31.05 29.26 27.62 26.13 ...
    24.76 23.50];
V_15 = [41.06 38.68 36.47 34.40 32.48 30.70 29.05 27.53 26.12 24.82 ...
    23.62 22.51];
V_20 = [37.59 35.63 33.77 32.03 30.40 28.87 27.45 26.12 24.88 23.73 ...
    22.66 21.66];
%
P_5  = [817 769 730 698 671 649 631 615 602 591 582 574];
P_10 = [782 746 715 689 667 648 632 618 606 595 586 579];
P_15 = [761 732 706 683 663 646 631 618 607 597 589 581];
P_20 = [747 722 699 679 661 645 632 620 609 599 591 584];
%
TP_5  = [3.16 2.95 2.75 2.57 2.40 2.25 2.12 1.99 1.88 1.78 1.68 1.60];
TP_10 = [2.91 2.73 2.57 2.41 2.27 2.14 2.02 1.91 1.81 1.72 1.63 1.55];
TP_15 = [2.72 2.57 2.42 2.29 2.16 2.04 1.94 1.84 1.75 1.66 1.58 1.51];
TP_20 = [2.58 2.44 2.31 2.18 2.07 1.97 1.87 1.78 1.69 1.61 1.54 1.47];
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Find the range at which payload is least sensitive to a change in %
%   the weight of power factor.                                      %
% Using interp1 function to get the best fit curves for the data    %
%   above.                                                           %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
R_int   = linspace(500,6000,1000);
%
P_int5  = interp1(R,P_5,R_int);
P_int10 = interp1(R,P_10,R_int);
P_int15 = interp1(R,P_15,R_int);
P_int20 = interp1(R,P_20,R_int);
```

```
%
P_max = zeros(1000,1);
P_min = zeros(1000,1);
P_del = zeros(1000,1);
% Find the minimum delta in payload
for i = 1:1000
  P_max(i) = max([P_int5(i),P_int10(i),P_int15(i),P_int20(i)]);
  P_min(i) = min([P_int5(i),P_int10(i),P_int15(i),P_int20(i)]);
  P_del(i) = P_max(i) - P_min(i);
  P_dm   = min(P_del);
end
% Find the corresponding range and payload to the minimum delta P
for i = 1:1000
  if P_del(i) <= P_dm;
    R_opt = R_int(i);
    P_opt = mean([P_int5(i),P_int10(i),P_int15(i),P_int20(i)]);
  end
end
% Find the range corresponding to a delta of less than 10 LT.
for i = 1:1000
  if P_del(i) > 10;
    R_min = R_int(i);
    P1_min = mean([P_int5(i),P_int10(i),P_int15(i),P_int20(i)]);
  end
end
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Plot the relationships from the data collected %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
figure('Position', [0 0 700 500]);
plot(R,V_5,R,V_10,'--',R,V_15,':',R,V_20,'--o','LineWidth',1.5)
legend('\beta = 5 lbs/hp','\beta = 10 lbs/hp','\beta = 15 lbs/hp', ...
    '\beta = 20 lbs/hp')
xlabel('Range (nm)')
ylabel('Speed (kts)')
xlim([0 6500])
title('Optimal Speed vs Range for Various Weight of Power Factor')
%
figure('Position', [0 0 700 500]);
plot(R,P_5,R,P_10,'--',R,P_15,':',R,P_20,'--o','LineWidth',1.5)
legend('\beta = 5 lbs/hp','\beta = 10 lbs/hp','\beta = 15 lbs/hp', ...
    '\beta = 20 lbs/hp')
xlabel('Range (nm)')
ylabel('Payload (LT)')
xlim([0 6500])
title('Optimal Payload vs Range for Various \beta (Refuel Rate = 2,000
gal/min)')
%
figure('Position', [0 0 700 500]);
plot(R,TP_5,R,TP_10,'--',R,TP_15,':',R,TP_20,'--o','LineWidth',1.5)
legend('\beta = 5 lbs/hp','\beta = 10 lbs/hp','\beta = 15 lbs/hp', ...
    '\beta = 20 lbs/hp')
xlabel('Range (nm)')
ylabel('Cargo Throughput (LT/hr)')
xlim([0 6500])
```

```
title('Optimal Throughput vs Range for Various Weight of Power Factor')
%
%%%%%%%%%%%%%%%%%%%%%
% Print the results %
%%%%%%%%%%%%%%%%%%%%%
fprintf('\nThe range that is least sensitive to changes in beta
(weight')
fprintf('\n  of power factor) is %4.0f nm,',R_opt)
fprintf('\n  with a payload variation of %1.1f LT,',P_dm)
fprintf('\n  and the average payload of %3.0f LT.\n',P_opt)
fprintf('\nThe minimum range at which the delta in payload become')
fprintf('\n  negligible (< 10 LT) is %4.0f nm,',R_min)
fprintf('\n  with an average payload of %3.0f LT\n\n',P1_min)
```

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APPENDIX L. MATLAB CODES: k SENSITIVITY ANALYSIS

```
%% SENSITIVITY ANALYSIS: REFUELING RATE
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% This program finds the effect of the refueling rate on the %
%   sensitivity of the weight of power factor.               %
% Displacement = 4,000 LT                                    %
% Refueling Rate = 1,000 gal/min and 4,000 gal/min           %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clc
clear all
close all
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Data from Optimization Program based on different value of beta %
%   R   = range (nm)                                                %
%   P1_i = optimal payload for beta equals i at 1,000 gal/min      %
%   P4_i = optimal payload for beta equals i at 4,000 gal/min      %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
R = [500 1000 1500 2000 2500 3000 3500 4000 4500 5000 5500 6000];
%
P1_5  = [868 823 785 751 723 698 677 659 643 629 617 607];
P1_10 = [835 799 767 739 714 693 674 657 643 630 619 609];
P1_15 = [812 781 753 729 707 687 670 655 641 629 619 609];
P1_20 = [795 768 743 721 701 683 668 653 641 629 619 610];
%
P4_5  = [757 719 682 654 631 612 597 585 575 567 560 554];
P4_10 = [736 703 675 652 632 616 602 591 582 573 566 560];
P4_15 = [721 693 670 650 633 618 606 595 585 577 570 564];
P4_20 = [711 688 667 649 634 620 608 598 589 581 574 568];
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Find the range at which payload is least sensitive to a change in %
%   the weight of power factor.                                      %
% Using interp1 function to get the best fit curves for the data     %
%   above.                                                           %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
R_int = linspace(500,6000,1000);
%
P1_int5  = interp1(R,P1_5,R_int);
P1_int10 = interp1(R,P1_10,R_int);
P1_int15 = interp1(R,P1_15,R_int);
P1_int20 = interp1(R,P1_20,R_int);
%
P4_int5  = interp1(R,P4_5,R_int);
P4_int10 = interp1(R,P4_10,R_int);
P4_int15 = interp1(R,P4_15,R_int);
P4_int20 = interp1(R,P4_20,R_int);
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Determine maximum change in payload at 3,500 nm %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
P1_del = P1_int5(546) - P1_int20(546);
P4_del = P4_int20(546) - P4_int5(546);
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Plot the relationships from the data collected %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
R_2 = 3500;
P_2 = linspace(500,900);
figure('Position', [0 0 700 700]);
plot(R,P1_5,R,P1_10,'-.',R,P1_15,'--x',R,P1_20,'--o',R,P4_5,'--b', ...
     R,P4_10,'--+g',R,P4_15,'--dr',R,P4_20,'--hc',R_2,P_2,':k', ...
     'LineWidth',1.5)
legend('\beta =  5 lbs/hp, 1,000 gal/min', ...
    '\beta = 10 lbs/hp, 1,000 gal/min', ...
    '\beta = 15 lbs/hp, 1,000 gal/min', ...
    '\beta = 20 lbs/hp, 1,000 gal/min', ...
    '\beta =  5 lbs/hp, 4,000 gal/min', ...
    '\beta = 10 lbs/hp, 4,000 gal/min', ...
    '\beta = 15 lbs/hp, 4,000 gal/min', ...
    '\beta = 20 lbs/hp, 4,000 gal/min', ...
    'R_{opt} for 2,000 gal/min')
xlabel('Range (nm)')
ylabel('Payload (LT)')
xlim([0 6500])
ylim([500 900])
title('Optimal Payload vs Range for Various \beta and Refuelling
Rates')
text(3600,680,'\DeltaP_{max} @ 3,500 nm = 9 LT')
text(1500,585,'\DeltaP_{max} @ 3,500 nm = 11 LT')
%
%%%%%%%%%%%%%%%%%%%%%
% Print the results %
%%%%%%%%%%%%%%%%%%%%%
fprintf('\nThe maximum difference in payload at 3,500 nm for a 1,000')
fprintf('\n  gal/min refueling rate is %1.0f LT\n',P1_del)
fprintf('\nThe maximum difference in payload at 3,500 nm for a 4,000')
fprintf('\n  gal/min refueling rate is %2.0f LT\n\n',P4_del)
```

APPENDIX M. MATLAB CODES: THROUGHPUT VERSUS DISTANCE

```
%% THROUGHPUT VS DELVIERY DISTANCE
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% This program plots the relationship between throughput and %
%   delivery distance for various design displacements.      %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clc
clear all
close all
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Data collected from the results of the MATLAB program      %
%   TP_Throughput_Optimization.m                             %
%   S - roundtrip delivery distance.                         %
%   TP - cargo throughput for different design displacement. %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
S1 = [ 500 1000 1335 1340 1500 2000 2500 3000 3500 4000 4500 5000 ...
    5500 6000 6500 7000];
S2 = [ 500 1000 1500 2000 2165 2170 2500 3000 3500 4000 4500 5000 ...
    5500 6000 6500 7000];
S3 = [ 500 1000 1500 2000 2500 2800 2805 3000 3500 4000 4500 5000 ...
    5500 6000 6500 7000];
S4 = [ 500 1000 1500 2000 2500 3000 3500   3505 4000 4500 5000 ...
    5500 6000 6500 7000];
S5 = [ 500 1000 1500 2000 2500 3000 3500 4000 4005   4500 5000 ...
    5500 6000 6500 7000];
S6 = [ 500 1000 1500 2000 2500 3000 3500 4000 4495   4500 5000 ...
    5500 6000 6500 7000];
%
TP1 = [6.16 4.20 3.46 3.26 3.00 2.41 2.01 1.73 1.51 1.34 1.21 1.10 ...
    1.01 0.93 0.87 0.81];
TP2 = [7.83 5.87 4.69 3.91 3.70 3.42 3.09 2.69 2.38 2.14 1.94 1.78 ...
    1.64 1.52 1.42 1.33];
TP3 = [8.67 6.83 5.64 4.80 4.18 3.88 3.54 3.37 3.01 2.72 2.48 2.28 ...
    2.11 1.97 1.84 1.73];
TP4 = [9.12 7.40 6.22 5.37 4.72 4.22 3.81   3.45 3.13 2.87 2.65 ...
    2.46 2.29 2.15 2.02];
TP5 = [9.47 7.86 6.72 5.87 5.21 4.68 4.25 3.89 3.51   3.23 2.98 ...
    2.78 2.59 2.44 2.30];
TP6 = [9.72 8.21 7.10 6.26 5.59 5.05 4.61 4.24 3.93   3.52 3.26 ...
    3.04 2.85 2.68 2.53];
%
%%%%%%%%%%%%%%%%%%%%
% Plot the results %
%%%%%%%%%%%%%%%%%%%%
figure('Position', [0 0 600 500]);
plot(S1,TP1,'--o',S2,TP2,'--x',S3,TP3,'-.',S4,TP4,'-',S5,TP5,'--', ...
     S6,TP6,':','LineWidth',1.5)
xlabel('Round-trip Delivery Distance (nm)')
ylabel('Cargo Throughput (LT/hr)')
```

```
title('Optimal Throughput vs Delivery Distance')
legend('\Delta = 1,000 LT','\Delta = 2,000 LT','\Delta = 3,000 LT',...
    '\Delta = 4,000 LT','\Delta = 5,000 LT','\Delta = 6,000 LT',1)
xlim([500 7000])
```

APPENDIX N. MATLAB CODES: THE OPTIMAL SOLUTIONS

```
%% OPTIMAL SOLUTIONS
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% This program outputs the graphical results of the optimal %
%   solutions.                                               %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clc
clear all
close all
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Data from optimal results:        %
%   D = Design Displacement (LT)    %
%   V = Design Optimal Speed (kts)  %
%   R = Design Optimal Range (nm)   %
%   P = Design Optimal Payload (LT) %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
D = [1000 2000 3000 4000 5000 6000];
V = [40.7 36.6 33.8 31.1 29.4 27.8];
R = [1337 2168 2801 3501 4002 4497];
P = [ 162  320  478  631  788  943];
%
%%%%%%%%%%%%%%%%%%%%
% Plot the results %
%%%%%%%%%%%%%%%%%%%%
figure('Position', [0 0 600 1000]);
subplot(311)
plot(D,R,'*')
xlabel('Design Displacement (LT)')
ylabel('Optimal Range (nm)')
title('Optimal Range vs Design Displacement')
xlim([0 7000])
%
subplot(312)
plot(D,V,'og')
xlabel('Design Displacement (LT)')
ylabel('Optimal Speed (kts)')
title('Optimal Speed vs Design Displacement')
xlim([0 7000])
%
subplot(313)
plot(D,P,'dr')
xlabel('Design Displacement (LT)')
ylabel('Optimal Payload (LT)')
title('Optimal Payload vs Design Displacement')
xlim([0 7000])
```

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LIST OF REFERENCES

[1] Military Sealift Command, “U.S. Navy’s Military Sealift Command.” [Online]. Available: www.msc.navy.mil/. [Accessed April 28, 2014].

[2] K. B. Artana and K. Ishida, “The determination of optimum ship’s design and power prediction using spreadsheet model,” Kobe University of Mercantile Marine, Kobe, Japan, 2003.

[3] C. B. McKesson, “A parametric method for characterizing the design space of high speed cargo ships,” Royal Institution of Naval Architects, London, UK, 2006.

[4] M. Yeh, “Throughput evaluation of an autonomous sustainment cargo container system,” M.S., Dept of Mech. and Astronat. Eng., Naval Postgraduate School, Monterey, CA, 2007.

[5] C. B. McKesson, “The utility of very simple models for very complex systems,” *SCS M&S Magazine*, vol. 11, no. 1, 23–29, January, 2011.

[6] A. S. Wright, “Physics based modeling of seabasing operations,” M.S., Dept of Mech. and Astronat. Eng., Naval Postgraduate School, Monterey, CA, 2011.

[7] *Wikipedia*, “Protector-class offshore patrol vessel.” [Online]. Available: http://en.wikipedia.org/wiki/Protector-class_offshore_patrol_vessel. [Accessed July 17, 2013].

[8] *Wikipedia*, “Takanami-class destroyer.” [Online]. Available: http://en.wikipedia.org/wiki/Takanami-class_destroyer. [Accessed July 17, 2013].

[9] *Wikipedia*, “Gwanggaeto the Great-class destroyer.” [Online]. Available: http://en.wikipedia.org/wiki/Gwanggaeto_the_Great-class_destroyer. [Accessed July 17, 2013].

[10] *Wikipedia*, “F-22P Zulfiquar-class frigate.” [Online]. Available: http://en.wikipedia.org/wiki/F-22P_Zulfiquar-class_frigate. [Accessed July 17, 2013].

[11] *Wikipedia*, “Niteroi-class frigate.” [Online]. Available: http://en.wikipedia.org/wiki/Niteroi-class_frigate. [Accessed July 17, 2013].

[12] *Wikipedia*, “MILGEM project.” [Online]. Available: http://en.wikipedia.org/wiki/Milgem-class_corvette. [Accessed July 17, 2013].

[13] *Wikipedia*, “USNS Guam (HST-1).” [Online]. Available: http://en.wikipedia.org/wiki/USNS_Guam_(HST-1). [Accessed July 17, 2013].

[14] *Wikipedia*, “MV Fairweather.” [Online]. Available: http://en.wikipedia.org/wiki/MV_Fairweather. [Accessed July 17, 2013].

[15] W. Spitzinger. (2013, July 28). “USS Independence (LCS-2) littoral combat ship / Corvette,” Military Factory. [Online]. Available: http://www.militaryfactory.com/searchresults.asp?q=LCS-2. [Accessed September 02, 2013].

[16] Naval-Technology, “Spearhead class joint high speed vessels (JHSV), United States of America.” [Online]. Available: http://www.naval-technology.com/projects/spearhead-class/. [Accessed July 17, 2013].

[17] J. Potts. (2013, September 9). “USS Freedom (LCS-1) littoral combat ship,” Military Factory. [Online]. Available: http://www.militaryfactory.com/ships/detail.asp?ship_id=USS-Freedom-LCS1. [Accessed July 17, 2013].

[18] IHS Jane’s. (2011, October 28). “Jane’s high-speed marine transportation: high-speed multihull vessels,” NQEA Australia Pty Ltd. [Online]. Available: https://janes.ihs.com/CustomPages/Janes/DisplayPage.aspx?DocType=Reference&ItemId=+++1358201&Pubabbrev=JHMT. [Accessed July 16, 2013].

[19] *Wikipedia*, “HSV-2 Swift.” [Online]. Available: http://en.wikipedia.org/wiki/HSV-2_Swift. [Accessed July 17, 2013].

[20] M. Mazumdar. (2012, April 9). “Indonesia launches first Klewang-class large trimaran missile boat.” [Online]. Available: http://www.defensemedianetwork.com/stories/indonesia-launches-new-class-of-large-trimaran-missile-boats/. [Accessed July 17, 2013].

[21] IHS Jane’s. (2011, March 29). “Jane’s high-speed marine transportation: high-speed multihull vessels,” Derecktor Shipyards. [Online]. Available: https://janes.ihs.com/CustomPages/Janes/DisplayPage.aspx?DocType=Reference&ItemId=+++1359157&Pubabbrev=JHMT. [Accessed July 16, 2013].

[22] A. M. Gorky, “Up-to-date fast passenger ferry of A45 project “Lena”,” PHC. [Online]. Available: http://www.zdship.ru/eproducts/shipbuilding/high-speed-vessels-construction/474/. [Accessed July 16, 2013].

[23] Gomel Shipyard. (2011, May 10). “Jane’s high-speed marine transportation: hydrofoils,” IHS Jane’s. [Online]. Available: https://janes.ihs.com/CustomPages/Janes/DisplayPage.aspx?DocType=Reference&ItemId=+++1358192&Pubabbrev=JHMT. [Accessed July 16, 2013].

[24] Boeing. (2011, July 4). “Jane’s high-speed marine transportation: hydrofoils,” IHS Jane’s. [Online]. Available: https://janes.ihs.com/CustomPages/Janes/DisplayPage.aspx?DocType=Reference&ItemId=+++1358195&Pubabbrev=JHMT. [Accessed July 16, 2013].

[25] Yong Choo Kui Shipyard Sdn Bhd. (2011, June 10). “Jane’s high-speed marine transportation: high-speed monohull craft,” IHS Jane’s. [Online]. Available: https://janes.ihs.com/CustomPages/Janes/DisplayPage.aspx?DocType=Reference&ItemId=+++1359221&Pubabbrev=JHMT. [Accessed July 16, 2013].

[26] S. Boyd and L. Vandenberghe. *Convex Optimization*. Cambridge, MA: Cambridge University Press, 2004.

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